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5 Modeling of FACTS in Three-Phase Power Flow and Three-Phase OPF Analysis

Three-phase power flow calculations are important tools to compute the realistic system operation states and evaluate the control performance of various control devices such as transformer, synchronous machines and FACTS-devices, particularly because (a) there are unbalances of three-phase transmission lines in high voltage transmission networks; (b) there are unbalanced three-phase loads; (c) in addition, there are one-phase or two-phase lines in some distribution networks, etc. Under these unbalanced operating conditions, three-phase power flow studies are needed to assess the realistic operating conditions of the systems and analyze the behavior and control performance of power system components including FACTS-devices.

A number of three-phase power flow methods such as Bus-Impedance Method [1], Newton-Raphson Method [2][3], Fast-Decoupled Method [4][6], Gauss-Seidel Method [5], Hybrid Method [7], A Newton approach combining representation of linear elements using linear nodal voltage equation and representation of nonlinear elements using injected currents and associated equality constraints [8], Implicit Bus-Impedance Method [9], Decoupling-Compensation Bus-Admittance Method [9], Fast Three-phase Load Flow Methods [10], and Newton power flow in current injection form [12] etc. have been proposed since 1960s. The Newton method proposed in [8] is in particular interfaced with EMTP (**E**lectro-**M**agnetic **T**ransients **P**rogram) and can be used to initialize the simulations. The Fast Three-phase Load Flow Methods proposed in [10] have been further implemented on a parallel processor [11].

In addition to the above three-phase power flow solution methods, specialized three-phase power flow techniques [13]-[21] for distribution networks have also been proposed with various success where the special structure of distribution networks is exploited and computational efficiency is improved. Modeling of power system components can be found in [22][23][6].

An Optimal Power Flow (OPF) program can be used to determine the optimal operation state of a power system by optimizing a particular objective while satisfying specified physical and operating constraints. Because of its capability of integrating the economic and secure aspects of the system into one mathematical model, the OPF can be applied not only in three-phase power system planning, but also in real time operation optimisation of three-phase power systems. With the incorporation of FACTS-devices into power systems, a three-phase optimal power flow will be required. In contrast to the research in three-phase power flow solu-

tion techniques, the research in optimal three-phase power flow methods has been very limited.

With the increasing installation of FACTS in power systems, modeling of FACTS-devices into three-phase power flow and optimal three-phase power flow analysis will be of great interest. In recent years, three-phase FACTS models have been investigated for three-phase power flow analysis [24][25]. Positive sequence models for FACTS-devices have been discussed in chapters 2, 3 and 4. However, three-phase FACTS models are more complex than those positive sequence ones since unbalanced conditions need to be considered. This chapter introduces the following aspects:

- review of three-phase power flow solution techniques;
- three-phase Newton power flow solution methods in polar and rectangular coordinates;
- three-phase FACTS models for SSSC and UPFC and their incorporation in three-phase power flow analysis;
- formulation of optimal three-phase power flow problems.

5.1 Three-Phase Newton Power Flow Methods in Rectangular Coordinates

Modeling of power system components such as transmission lines, loads, etc. have been discussed in [23][6]. In the following, the formulation of three-phase Newton power flow in rectangular coordinates will be presented where the modeling of synchronous generator is discussed in detail.

5.1.1 Classification of Buses

In three-phase power flow calculations, all buses may be classified into the following categories:

Slack bus. Similar to that in single-phase positive-sequence power flow calculations, a slack bus, which is usually one of the generator terminal buses, should be selected for three-phase power flow calculations. At the slack bus, the positive-sequence voltage angle and magnitude are specified while the active and reactive power injections at the generator terminal are unknown. The voltage angle of the slack bus is taken as the reference for the angles of all other buses. Usually there is only one slack bus in a system. However, in some production grade programs, it may be possible to include more than one bus as distributed slack buses.

PV Buses. PV buses in three-phase power flow calculations are usually generator terminal buses. For these buses, the total active power injections and positive-sequence voltage magnitudes are specified.

PQ Buses. PQ buses are usually load buses in the network. For these buses, the active and reactive power injections of their three-phases are specified.

5.1.2 Representation of Synchronous Machines

A synchronous machine may be represented by a set of three-phase balanced voltage sources in series with a 3 by 3 impedance matrix. Such a synchronous machine model is shown in Fig. 5.1. The impedance matrix $\mathbf{Zg}_i$ may be determined by positive-, negative-, and zero-sequence impedance parameters of a synchronous machine. $\mathbf{Zg}_i$ is defined in Appendix A of this chapter.

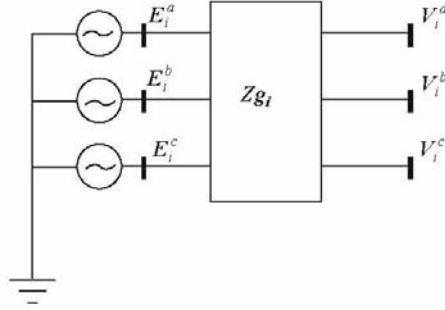


Fig. 5.1. A synchronous machine

It is assumed that the synchronous generator in Fig. 5.1 has a round rotor structure, and saturation of the synchronous generator is not considered in the present model. However, in principle, there is no difficulty to take into account the saturation.

In Fig. 5.1, $V_i^a = E_i^a + jF_i^a$, $V_i^b = E_i^b + jF_i^b$, $V_i^c = E_i^c + jF_i^c$, which are the three-phase voltages at the generator terminal bus, are expressed in phasors in rectangular coordinates. Similarly, the voltages at the generator internal bus may be given by $E_i^a = Eg_i^a + jFg_i^a$, $E_i^b = Eg_i^b + jFg_i^b$, $E_i^c = Eg_i^c + jFg_i^c$. In fact the voltages at the generator internal bus are balanced, that is:

$$E_i^b = E_i^a e^{-j2\pi/3} \quad (5.1)$$

$$E_i^c = E_i^a e^{j2\pi/3} \quad (5.2)$$

In the three-phase power flow equations of the generator, Eg_i^a and Fg_i^a can be considered as independent state variables of the internal generator bus while Eg_i^b and Fg_i^b , and Eg_i^c and Fg_i^c are dependent state variables and can be represented by Eg_i^a and Fg_i^a . We have:

$$Eg_i^b = -\frac{1}{2}Eg_i^a + \frac{\sqrt{3}}{2}Fg_i^a \quad (5.3)$$

$$Fg_i^b = -\frac{\sqrt{3}}{2}Eg_i^a - \frac{1}{2}Fg_i^a \quad (5.4)$$

$$Eg_i^c = -\frac{1}{2}Eg_i^a - \frac{\sqrt{3}}{2}Fg_i^a \quad (5.5)$$

$$Fg_i^c = -\frac{\sqrt{3}}{2}Eg_i^a - \frac{1}{2}Fg_i^a \quad (5.6)$$

5.1.3 Power and Voltage Mismatch Equations in Rectangular Coordinates

5.1.3.1 Power Mismatch Equations at Network Buses

The network buses include all buses of the network except the internal buses of generators. The power mismatch equations of phase p at the network bus i are given by:

$$\Delta P_i^p = -Pd_i^p - \sum_{j \in i} [E_i^p (G_{ij}^{pm} E_j^m - B_{ij}^{pm} F_j^m) + F_i^p (G_{ij}^{pm} F_j^m + B_{ij}^{pm} E_j^m)] \quad (5.7)$$

$$\Delta Q_i^p = -Qd_i^p - \sum_{j \in i} [F_i^p (G_{ij}^{pm} E_j^m - B_{ij}^{pm} F_j^m) - E_i^p (G_{ij}^{pm} F_j^m + B_{ij}^{pm} E_j^m)] \quad (5.8)$$

where $p = a, b, c$. Pd_i^p and Qd_i^p are the active and reactive loads of phase p at bus i .

5.1.3.2 Power and Voltage Mismatch Equations of Synchronous Machines

PQ Machines. For a PQ machine, the total three-phase active and reactive powers at the terminal bus of the machine are specified:

$$\begin{aligned} \Delta P_{g_i} &= -P_{g_i}^{Spec} \\ &- \sum_{p=a,b,c} \sum_{m=a,b,c} [E_i^p (G_{gi}^{pm} E_i^m - B_{gi}^{pm} F_i^m) + F_i^p (G_{gi}^{pm} F_i^m + B_{gi}^{pm} E_i^m)] \\ &+ \sum_{p=a,b,c} \sum_{m=a,b,c} [E_i^p (G_{gi}^{pm} E_g^m - B_{gi}^{pm} F_g^m) + F_i^p (G_{gi}^{pm} F_g^m + B_{gi}^{pm} E_g^m)] \end{aligned} \quad (5.9)$$

$$\begin{aligned} \Delta Q_{g_i} &= -Q_{g_i}^{Spec} \\ &- \sum_{p=a,b,c} \sum_{m=a,b,c} [F_i^p (G_{gi}^{pm} E_i^m - B_{gi}^{pm} F_i^m) - E_i^p (G_{gi}^{pm} F_i^m + B_{gi}^{pm} E_i^m)] \\ &+ \sum_{p=a,b,c} \sum_{m=a,b,c} [F_i^p (G_{gi}^{pm} E_g^m - B_{gi}^{pm} F_g^m) - E_i^p (G_{gi}^{pm} F_g^m + B_{gi}^{pm} E_g^m)] \end{aligned} \quad (5.10)$$

where Pg_i^{Spec} and Qg_i^{Spec} are the specified active and reactive powers of the generator at bus i , which are in the direction of terminal bus i .

PV Machines. For a PV machine, the total three-phase active power flow and the positive sequence voltage magnitude at its terminal bus i are specified. The active power flow mismatch equation is given by (5.9) while the voltage mismatch equation at bus i is given by:

$$\Delta Vg_i = V_i^{Spec} - V_i^1 = V_i^{Spec} - \sqrt{(e_i^1)^2 + (f_i^1)^2} \quad (5.11)$$

where V_i^1 is the positive-sequence voltage magnitude voltage at the generator terminal bus i . e_i^1 and f_i^1 are the real and imaginary parts of the positive-sequence voltage phasor at bus i and they are given by:

$$e_i^1 = \text{Re}(V_i^a + V_i^b e^{j120^\circ} + V_i^c e^{j240^\circ}) / 3 \quad (5.12)$$

$$f_i^1 = \text{Im}(V_i^a + V_i^b e^{j120^\circ} + V_i^c e^{j240^\circ}) / 3 \quad (5.13)$$

where V_i^a , V_i^b and V_i^c are the phase a , phase b and phase c voltages at bus i , respectively.

Slack Machine. At the terminal bus of the Slack machine, the positive-sequence voltage magnitude is specified and the positive-sequence voltage angle is taken as the system reference. We have:

$$\Delta \theta g_i = f_i^1 = 0 \quad (5.14)$$

$$\Delta Vg_i = V_i^{Spec} - V_i^1 = V_i^{Spec} - \sqrt{(e_i^1)^2 + (f_i^1)^2} \quad (5.15)$$

where V_i^{Spec} is the specified positive-sequence voltage at the terminal bus of the slack machine. e_i^1 and f_i^1 are the real and imaginary parts of the positive-sequence voltage at the terminal bus of the Slack machine, and they are defined in (5.11) and (5.12).

5.1.4 Formulation of Newton Equations in Rectangular Coordinates

Combining the power mismatch equations of network buses and generator active power and voltage control constraints for the case of PV machines, the following Newton equation in rectangular coordinates can be obtained:

$$\mathbf{J} \Delta \mathbf{X} = -\mathbf{F}(\mathbf{X}) \quad (5.16)$$

where $\Delta \mathbf{X} = [\Delta \mathbf{X}_{gen}, \Delta \mathbf{X}_{sys}]^T$

$$\begin{aligned}
\Delta \mathbf{X}_{gen} &= [\Delta E_i^a, \Delta F_i^a]^T \\
\Delta \mathbf{X}_{sys} &= [\Delta E_i^a, \Delta F_i^a, \Delta E_i^b, \Delta F_i^b, \Delta E_i^c, \Delta F_i^c, \Delta E_j^a, \Delta F_j^a, \Delta E_j^b, \Delta F_j^b, \Delta E_j^c, \Delta F_j^c]^T \\
\mathbf{F}(\mathbf{X}) &= [\mathbf{F}_{gen}, \mathbf{F}_{sys}]^T \\
\mathbf{F}_{gen} &= [f_{gen}^1, f_{gen}^2]^T \\
\mathbf{F}_{sys} &= [\Delta P_i^a, \Delta Q_i^a, \Delta P_i^b, \Delta Q_i^b, \Delta P_i^c, \Delta Q_i^c, \Delta P_j^a, \Delta Q_j^a, \Delta P_j^b, \Delta Q_j^b, \Delta P_j^c, \Delta Q_j^c]^T \\
\mathbf{J} &= \frac{\partial \mathbf{F}(\mathbf{X})}{\partial \mathbf{X}}
\end{aligned}$$

The Jacobian elements of the network block are defined as:

$$\frac{\partial \Delta P_i^p}{\partial E_j^m} = \begin{cases} -(G_{ij}^{pm} E_i^p + B_{ij}^{pm} F_i^p) & (j \neq i, \text{ or } m \neq p) \\ -\sum_{j \in i} \sum_{m=a,b,c} (G_{ij}^{pm} E_j^m - B_{ij}^{pm} F_j^m) - G_{ii}^{pp} E_i^p - B_{ii}^{pp} F_i^p & (j = i, m = p) \end{cases} \quad (5.17)$$

$$\frac{\partial \Delta P_i^p}{\partial F_j^m} = \begin{cases} B_{ij}^{pm} E_i^p - G_{ij}^{pm} F_i^p & (j \neq i, \text{ or } m \neq p) \\ -\sum_{j \in i} \sum_{m=a,b,c} (G_{ij}^{pm} F_j^m + B_{ij}^{pm} E_j^m) + B_{ii}^{pp} E_i^p - G_{ii}^{pp} F_i^p & (j = i, m = p) \end{cases} \quad (5.18)$$

$$\frac{\partial \Delta Q_i^p}{\partial E_j^m} = \begin{cases} B_{ij}^{pm} E_i^p - G_{ij}^{pm} F_i^p & (j \neq i, \text{ or } m \neq p) \\ -\sum_{j \in i} \sum_{m=a,b,c} (G_{ij}^{pm} F_j^m + B_{ij}^{pm} E_j^m) + B_{ii}^{pp} E_i^p - G_{ii}^{pp} F_i^p & (j = i, m = p) \end{cases} \quad (5.19)$$

$$\frac{\partial \Delta Q_i^p}{\partial F_j^m} = \begin{cases} G_{ij}^{pm} E_i^p + B_{ij}^{pm} F_i^p & (j \neq i, \text{ or } m \neq p) \\ -\sum_{j \in i} \sum_{m=a,b,c} (G_{ij}^{pm} E_j^m - B_{ij}^{pm} F_j^m) + G_{ii}^{pp} E_i^p + B_{ii}^{pp} F_i^p & (j = i, m = p) \end{cases} \quad (5.20)$$

In addition, we can find the following partial differentials with respect to generator internal variables E_i^m, F_i^m ($m = a, b, c$):

$$\frac{\partial \Delta P_i^p}{\partial E_i^m} = (G_{ii}^{pm} E_i^p + B_{ii}^{pm} F_i^p) \quad (5.21)$$

$$\frac{\partial \Delta P_i^p}{\partial F_i^m} = -B_{ii}^{pm} E_i^p + G_{ii}^{pm} F_i^p \quad (5.22)$$

$$\frac{\partial \Delta Q_i^p}{\partial E_i^m} = -B g_i^{pm} E_i^p + G g_i^{pm} F_i^p \quad (5.23)$$

$$\frac{\partial \Delta Q_i^p}{\partial F_i^m} = -G g_i^{pm} E_i^p - B g_i^{pm} F_i^p \quad (5.24)$$

Assuming $P g_i^p$ and $Q g_i^p$ are the active and reactive generator output of phase p at the terminal bus i , we have $P g_i = \sum_{p=a,b,c} P g_i^p$ and $Q g_i = \sum_{p=a,b,c} Q g_i^p$. Following

the above formulas, we can find $\frac{\partial \Delta P g_i^p}{\partial E_i^m}$, $\frac{\partial \Delta P g_i^p}{\partial F_i^m}$, $\frac{\partial \Delta P g_i^p}{\partial E_i^m}$, $\frac{\partial \Delta P g_i^p}{\partial F_i^m}$, $\frac{\partial \Delta Q g_i^p}{\partial E_i^m}$,

$\frac{\partial \Delta Q g_i^p}{\partial E_i^m}$, $\frac{\partial \Delta Q g_i^p}{\partial F_i^m}$, $\frac{\partial \Delta Q g_i^p}{\partial F_i^m}$. The differentials of the synchronous machine

power mismatches with respect to the internal voltage variables E_i^m , F_i^m ($m = a, b, c$) are given by:

$$\frac{\partial \Delta P g_i}{\partial E_i^m} = \sum_{p=a,b,c} \frac{\partial \Delta P g_i^p}{\partial E_i^m} \quad (5.25)$$

$$\frac{\partial \Delta P g_i}{\partial F_i^m} = \sum_{p=a,b,c} \frac{\partial \Delta P g_i^p}{\partial F_i^m} \quad (5.26)$$

$$\frac{\partial \Delta P g_i}{\partial E_i^m} = \sum_{p=a,b,c} \frac{\partial \Delta P g_i^p}{\partial E_i^m} \quad (5.27)$$

$$\frac{\partial \Delta P g_i}{\partial F_i^m} = \sum_{p=a,b,c} \frac{\partial \Delta P g_i^p}{\partial F_i^m} \quad (5.28)$$

$$\frac{\partial \Delta Q g_i}{\partial E_i^m} = \sum_{p=a,b,c} \frac{\partial \Delta Q g_i^p}{\partial E_i^m} \quad (5.29)$$

$$\frac{\partial \Delta Q g_i}{\partial F_i^m} = \sum_{p=a,b,c} \frac{\partial \Delta Q g_i^p}{\partial F_i^m} \quad (5.30)$$

$$\frac{\partial \Delta Q g_i}{\partial E_i^m} = \sum_{p=a,b,c} \frac{\partial \Delta Q g_i^p}{\partial E_i^m} \quad (5.31)$$

$$\frac{\partial \Delta Q g_i}{\partial F_i^m} = \sum_{p=a,b,c} \frac{\partial \Delta Q g_i^p}{\partial F_i^m} \quad (5.32)$$

where $m = a, b, c$.

As mentioned, actually in the three-phase power flow equations of the generator, Eg_i^a and Fg_i^a can be considered as independent state variables of the internal generator bus while Eg_i^b and Fg_i^b , and Eg_i^c and Fg_i^c are dependent state variables and can be represented by Eg_i^a and Fg_i^a . We have:

$$\begin{aligned} \frac{\partial \Delta P g_i}{\partial E g_i^a} = & \sum_{p=a,b,c} \frac{\partial \Delta P g_i^p}{\partial E g_i^a} \\ & + \sum_{p=a,b,c} \frac{\partial \Delta P g_i^p}{\partial E g_i^b} \frac{\partial E g_i^b}{\partial E g_i^a} + \sum_{p=a,b,c} \frac{\partial \Delta P g_i^p}{\partial F g_i^b} \frac{\partial F g_i^b}{\partial E g_i^a} \\ & + \sum_{p=a,b,c} \frac{\partial \Delta P g_i^p}{\partial E g_i^c} \frac{\partial E g_i^c}{\partial E g_i^a} + \sum_{p=a,b,c} \frac{\partial \Delta P g_i^p}{\partial F g_i^c} \frac{\partial F g_i^c}{\partial E g_i^a} \end{aligned} \quad (5.33)$$

$$\begin{aligned} \frac{\partial \Delta P g_i}{\partial F g_i^a} = & \sum_{p=a,b,c} \frac{\partial \Delta P g_i^p}{\partial F g_i^a} \\ & + \sum_{p=a,b,c} \frac{\partial \Delta P g_i^p}{\partial E g_i^b} \frac{\partial E g_i^b}{\partial F g_i^a} + \sum_{p=a,b,c} \frac{\partial \Delta P g_i^p}{\partial F g_i^b} \frac{\partial F g_i^b}{\partial F g_i^a} \\ & + \sum_{p=a,b,c} \frac{\partial \Delta P g_i^p}{\partial E g_i^c} \frac{\partial E g_i^c}{\partial F g_i^a} + \sum_{p=a,b,c} \frac{\partial \Delta P g_i^p}{\partial F g_i^c} \frac{\partial F g_i^c}{\partial F g_i^a} \end{aligned} \quad (5.34)$$

$$\begin{aligned} \frac{\partial \Delta Q g_i}{\partial E g_i^a} = & \sum_{p=a,b,c} \frac{\partial \Delta Q g_i^p}{\partial E g_i^a} \\ & + \sum_{p=a,b,c} \frac{\partial \Delta Q g_i^p}{\partial E g_i^b} \frac{\partial E g_i^b}{\partial E g_i^a} + \sum_{p=a,b,c} \frac{\partial \Delta Q g_i^p}{\partial F g_i^b} \frac{\partial F g_i^b}{\partial E g_i^a} \\ & + \sum_{p=a,b,c} \frac{\partial \Delta Q g_i^p}{\partial E g_i^c} \frac{\partial E g_i^c}{\partial E g_i^a} + \sum_{p=a,b,c} \frac{\partial \Delta Q g_i^p}{\partial F g_i^c} \frac{\partial F g_i^c}{\partial E g_i^a} \end{aligned} \quad (5.35)$$

$$\begin{aligned} \frac{\partial \Delta Q g_i}{\partial F g_i^a} = & \sum_{p=a,b,c} \frac{\partial \Delta Q g_i^p}{\partial F g_i^a} \\ & + \sum_{p=a,b,c} \frac{\partial \Delta Q g_i^p}{\partial E g_i^b} \frac{\partial E g_i^b}{\partial F g_i^a} + \sum_{p=a,b,c} \frac{\partial \Delta Q g_i^p}{\partial F g_i^b} \frac{\partial F g_i^b}{\partial F g_i^a} \\ & + \sum_{p=a,b,c} \frac{\partial \Delta Q g_i^p}{\partial E g_i^c} \frac{\partial E g_i^c}{\partial F g_i^a} + \sum_{p=a,b,c} \frac{\partial \Delta Q g_i^p}{\partial F g_i^c} \frac{\partial F g_i^c}{\partial F g_i^a} \end{aligned} \quad (5.36)$$

Using the relationships in (5.3)-(5.6), (5.25)-(5.36) can be simplified as:

$$\begin{aligned} \frac{\partial \Delta P_{g_i}}{\partial E_{g_i}^a} = & \sum_{p=a,b,c} \frac{\partial \Delta P_{g_i}^p}{\partial E_{g_i}^a} \\ & - \frac{1}{2} \sum_{p=a,b,c} \frac{\partial \Delta P_{g_i}^p}{\partial E_{g_i}^b} - \frac{\sqrt{3}}{2} \sum_{p=a,b,c} \frac{\partial \Delta P_{g_i}^p}{\partial F_{g_i}^b} \\ & - \frac{1}{2} \sum_{p=a,b,c} \frac{\partial \Delta P_{g_i}^p}{\partial E_{g_i}^c} - \frac{\sqrt{3}}{2} \sum_{p=a,b,c} \frac{\partial \Delta P_{g_i}^p}{\partial F_{g_i}^c} \end{aligned} \quad (5.37)$$

$$\begin{aligned} \frac{\partial \Delta P_{g_i}}{\partial F_{g_i}^a} = & \sum_{p=a,b,c} \frac{\partial \Delta P_{g_i}^p}{\partial F_{g_i}^a} \\ & + \frac{\sqrt{3}}{2} \sum_{p=a,b,c} \frac{\partial \Delta P_{g_i}^p}{\partial E_{g_i}^b} - \frac{1}{2} \sum_{p=a,b,c} \frac{\partial \Delta P_{g_i}^p}{\partial F_{g_i}^b} \\ & - \frac{\sqrt{3}}{2} \sum_{p=a,b,c} \frac{\partial \Delta P_{g_i}^p}{\partial E_{g_i}^c} - \frac{1}{2} \sum_{p=a,b,c} \frac{\partial \Delta P_{g_i}^p}{\partial F_{g_i}^c} \end{aligned} \quad (5.38)$$

$$\begin{aligned} \frac{\partial \Delta Q_{g_i}}{\partial E_{g_i}^a} = & \sum_{p=a,b,c} \frac{\partial \Delta Q_{g_i}^p}{\partial E_{g_i}^a} \\ & - \frac{1}{2} \sum_{p=a,b,c} \frac{\partial \Delta Q_{g_i}^p}{\partial E_{g_i}^b} - \frac{\sqrt{3}}{2} \sum_{p=a,b,c} \frac{\partial \Delta Q_{g_i}^p}{\partial F_{g_i}^b} \\ & - \frac{1}{2} \sum_{p=a,b,c} \frac{\partial \Delta Q_{g_i}^p}{\partial E_{g_i}^c} - \frac{\sqrt{3}}{2} \sum_{p=a,b,c} \frac{\partial \Delta Q_{g_i}^p}{\partial F_{g_i}^c} \end{aligned} \quad (5.39)$$

$$\begin{aligned} \frac{\partial \Delta Q_{g_i}}{\partial F_{g_i}^a} = & \sum_{p=a,b,c} \frac{\partial \Delta Q_{g_i}^p}{\partial F_{g_i}^a} \\ & + \frac{\sqrt{3}}{2} \sum_{p=a,b,c} \frac{\partial \Delta Q_{g_i}^p}{\partial E_{g_i}^b} - \frac{1}{2} \sum_{p=a,b,c} \frac{\partial \Delta Q_{g_i}^p}{\partial F_{g_i}^b} \\ & - \frac{\sqrt{3}}{2} \sum_{p=a,b,c} \frac{\partial \Delta Q_{g_i}^p}{\partial E_{g_i}^c} - \frac{1}{2} \sum_{p=a,b,c} \frac{\partial \Delta Q_{g_i}^p}{\partial F_{g_i}^c} \end{aligned} \quad (5.40)$$

Similarly, if Eg_i^a and Fg_i^a can be considered as independent state variables of the internal generator bus while Eg_i^b and Fg_i^b , and Eg_i^c and Fg_i^c are dependent state variables and can be represented by Eg_i^a and Fg_i^a , then we have

$$\begin{aligned} \frac{\partial \Delta P_i^p}{\partial Eg_i^a} = & (Gg_i^{pa} E_i^p + Bg_i^{pa} F_i^p) \\ & - \frac{1}{2} (Gg_i^{pb} E_i^p + Bg_i^{pb} F_i^p) - \frac{\sqrt{3}}{2} (-Bg_i^{pb} E_i^p + Gg_i^{pb} F_i^p) \\ & - \frac{1}{2} (Gg_i^{pc} E_i^p + Bg_i^{pc} F_i^p) - \frac{\sqrt{3}}{2} (-Bg_i^{pc} E_i^p + Gg_i^{pc} F_i^p) \end{aligned} \quad (5.41)$$

$$\begin{aligned} \frac{\partial \Delta P_i^p}{\partial Fg_i^a} = & -Bg_i^{pa} E_i^p + Gg_i^{pa} F_i^p \\ & + \frac{\sqrt{3}}{2} (Gg_i^{pb} E_i^p + Bg_i^{pb} F_i^p) - \frac{1}{2} (-Bg_i^{pb} E_i^p + Gg_i^{pb} F_i^p) \\ & - \frac{\sqrt{3}}{2} (Gg_i^{pc} E_i^p + Bg_i^{pc} F_i^p) - \frac{1}{2} (-Bg_i^{pc} E_i^p + Gg_i^{pc} F_i^p) \end{aligned} \quad (5.42)$$

$$\begin{aligned} \frac{\partial \Delta Q_i^p}{\partial Eg_i^a} = & -Bg_i^{pa} E_i^p + Gg_i^{pa} F_i^p \\ & - \frac{1}{2} (-Bg_i^{pb} E_i^p + Gg_i^{pb} F_i^p) - \frac{\sqrt{3}}{2} (-Gg_i^{pb} E_i^p - Bg_i^{pb} F_i^p) \\ & - \frac{1}{2} (-Bg_i^{pc} E_i^p + Gg_i^{pc} F_i^p) - \frac{\sqrt{3}}{2} (-Gg_i^{pc} E_i^p - Bg_i^{pc} F_i^p) \end{aligned} \quad (5.43)$$

$$\begin{aligned} \frac{\partial \Delta Q_i^p}{\partial Fg_i^a} = & -Gg_i^{pa} E_i^p - Bg_i^{pa} F_i^p \\ & + \frac{\sqrt{3}}{2} (-Bg_i^{pb} E_i^p + Gg_i^{pb} F_i^p) - \frac{1}{2} (-Gg_i^{pb} E_i^p - Bg_i^{pb} F_i^p) \\ & - \frac{\sqrt{3}}{2} (-Bg_i^{pc} E_i^p + Gg_i^{pc} F_i^p) - \frac{1}{2} (-Gg_i^{pc} E_i^p - Bg_i^{pc} F_i^p) \end{aligned} \quad (5.44)$$

5.2 Three-Phase Newton Power Flow Methods in Polar Coordinates

5.2.1 Representation of Generators

In Fig. 5.1, $V_i^a = V_i^a \angle \theta_i^a$, $V_i^b = V_i^b \angle \theta_i^b$, $V_i^c = V_i^c \angle \theta_i^c$, which are the three-phase voltages at the generator terminal bus, are expressed in phasors in rectangular coordinates. Similarly, the voltages at the generator internal bus may be given by $E_i^a = E_i^a \angle \delta_i^a$, $E_i^b = E_i^b \angle \delta_i^b$, $E_i^c = E_i^c \angle \delta_i^c$. In fact the voltages at the generator internal bus are balanced, that is:

$$E_i^b = E_i^a e^{-j2\pi/3} \quad (5.45)$$

$$E_i^c = E_i^a e^{j2\pi/3} \quad (5.46)$$

In the three-phase power flow equations of the generator, E_i^a and δ_i^a can be considered as independent state variables of the internal generator bus while E_i^b and δ_i^b , and E_i^c and δ_i^c are dependent state variables and can be represented by E_i^a and δ_i^a . We have:

$$E_i^b = E_i^a \quad (5.47)$$

$$\delta_i^b = \delta_i^a - \frac{2\pi}{3} \quad (5.48)$$

$$E_i^c = E_i^a \quad (5.49)$$

$$\delta_i^c = \delta_i^a + \frac{2\pi}{3} \quad (5.50)$$

5.2.2 Power and Voltage Mismatch Equations in Polar Coordinates

5.2.2.1 Power Mismatch Equations at Network Buses

The network buses include all buses of the network except the internal buses of generators. The power mismatch equation of phase p at the network bus i are given by:

$$\Delta P_i^p = -Pd_i^p - V_i^p \sum_{j \in i} \sum_{m=a,b,c} V_j^m (G_{ij}^{pm} \cos \theta_{ij}^{pm} + B_{ij}^{pm} \sin \theta_{ij}^{pm}) \quad (5.51)$$

$$\Delta Q_i^p = -Qd_i^p - V_i^p \sum_{j \in i} \sum_{m=a,b,c} V_j^m (G_{ij}^{pm} \sin \theta_{ij}^{pm} - B_{ij}^{pm} \cos \theta_{ij}^{pm}) \quad (5.52)$$

where $p = a, b, c$. Pd_i^p and Qd_i^p are the active and reactive loads of phase p at bus i .

5.2.2.2 Power and Voltage Mismatch Equations of Synchronous Machines

PQ Machines. For a PQ machine, the total three-phase active and reactive powers at the terminal bus of the machine are specified:

$$\begin{aligned} \Delta P g_i &= -P g_i^{Spec} \\ &- \sum_{p=a,b,c} \sum_{m=a,b,c} [V_i^p V_i^m (G g_i^{pm} \cos \theta_i^{pm} + B g_i^{pm} \sin \theta_i^{pm})] \\ &+ \sum_{p=a,b,c} \sum_{m=a,b,c} [V_i^p E_i^p (G g_i^{pm} \cos(\theta_i^p - \delta_i^m) + B g_i^{pm} \sin(\theta_i^p - \delta_i^m))] \end{aligned} \quad (5.53)$$

$$\begin{aligned} \Delta Q g_i &= -Q g_i^{Spec} \\ &- \sum_{p=a,b,c} \sum_{m=a,b,c} [V_i^p V_i^m (G g_i^{pm} \sin \theta_i^{pm} - B g_i^{pm} \cos \theta_i^{pm})] \\ &+ \sum_{p=a,b,c} \sum_{m=a,b,c} [V_i^p E_i^p (G g_i^{pm} \sin(\theta_i^p - \delta_i^m) - B g_i^{pm} \cos(\theta_i^p - \delta_i^m))] \end{aligned} \quad (5.54)$$

where $P g_i^{Spec}$ and $Q g_i^{Spec}$ are the specified active and reactive powers of the generator at bus i , which are in the direction of into terminal bus i .

PV Machines. For a PV machine, the total three-phase active power flow and the positive sequence voltage magnitude at its terminal bus i are specified. The active power flow mismatch equation is given by (5.9) while the voltage mismatch equation at bus i is given by:

$$\Delta V g_i = V_i^{Spec} - V_i^1 = V_i^{Spec} - \sqrt{(e_i^1)^2 + (f_i^1)^2} \quad (5.55)$$

where V_i^1 is the positive-sequence voltage magnitude voltage at the generator terminal bus i . e_i^1 and f_i^1 are the real and imaginary parts of the positive-sequence voltage phasor at bus i and they are given by (5.11) and (5.12)

Slack Machine. At the terminal bus of the Slack machine, the positive-sequence voltage magnitude is specified and the positive-sequence voltage angle is taken as the system reference. We have:

$$\Delta \theta g_i = f_i^1 = 0 \quad (5.56)$$

$$\Delta V_{g_i} = V_i^{Spec} - V_i^1 = V_i^{Spec} - \sqrt{(e_i^1)^2 + (f_i^1)^2} \quad (5.57)$$

where V_i^{Spec} is the specified positive-sequence voltage at the terminal bus of the slack machine. e_i^1 and f_i^1 are the real and imaginary parts of the positive-sequence voltage at the terminal bus of the Slack machine, and they are defined in (5.11) and (5.12).

5.2.3 Formulation of Newton Equations in Polar Coordinates

Combining the power mismatch equations of network buses and generator active power and voltage control constraints for the case of PV machines, the following Newton equation in polar coordinates can be obtained:

$$\mathbf{J}\Delta\mathbf{X} = -\mathbf{F}(\mathbf{X}) \quad (5.58)$$

where $\Delta\mathbf{X} = [\Delta\mathbf{X}_{gen}, \Delta\mathbf{X}_{sys}]^T$

$$\Delta\mathbf{X}_{gen} = [\Delta\delta_i^a, \Delta E_i^a]^T$$

$$\Delta\mathbf{X}_{sys} = [\Delta\theta_i^a, \Delta V_i^a, \Delta\theta_i^b, \Delta V_i^b, \Delta\theta_i^c, \Delta V_i^c, \Delta\theta_j^a, \Delta V_j^a, \Delta\theta_j^b, \Delta V_j^b, \Delta\theta_j^c, \Delta V_j^c]^T$$

$$\mathbf{F}(\mathbf{X}) = [\mathbf{F}_{gen}, \mathbf{F}_{sys}]^T$$

$$\mathbf{F}_{gen} = [f_{gen}^1, f_{gen}^2]^T$$

$$\mathbf{F}_{sys} = [\Delta P_i^a, \Delta Q_i^a, \Delta P_i^b, \Delta Q_i^b, \Delta P_i^c, \Delta Q_i^c, \Delta P_j^a, \Delta Q_j^a, \Delta P_j^b, \Delta Q_j^b, \Delta P_j^c, \Delta Q_j^c]^T$$

$$\mathbf{J} = \frac{\partial \mathbf{F}(\mathbf{X})}{\partial \mathbf{X}}$$

The Jacobian elements are defined as:

$$\frac{\partial \Delta P_i^p}{\partial \theta_j^m} = \begin{cases} -V_i^p V_j^m (G_{ij}^{pm} \sin \theta_{ij}^{pm} - B_{ij}^{pm} \cos \theta_{ij}^{pm}) & (j \neq i, m \neq p) \\ Q_i^p + (V_i^p)^2 B_{ii}^{pp} & (j = i, m = p) \end{cases} \quad (5.59)$$

$$\frac{\partial \Delta P_i^p}{\partial V_j^m} = \begin{cases} -V_i^p (G_{ij}^{pm} \cos \theta_{ij}^{pm} + B_{ij}^{pm} \sin \theta_{ij}^{pm}) & (j \neq i, m \neq p) \\ V_i^p B_{ii}^{pp} - P_i^p / V_i^p & (j = i, m = p) \end{cases} \quad (5.60)$$

$$\frac{\partial \Delta Q_i^p}{\partial \theta_j^m} = \begin{cases} V_i^p V_j^m (G_{ij}^{pm} \cos \theta_{ij}^{pm} + B_{ij}^{pm} \sin \theta_{ij}^{pm}) & (j \neq i, m \neq p) \\ -P_i^p + (V_i^p)^2 G_{ii}^{pp} & (j = i, m = p) \end{cases} \quad (5.61)$$

$$\frac{\partial \Delta Q_i^p}{\partial V_j^m} = \begin{cases} -V_i^p (G_{ij}^{pm} \sin \theta_{ij}^{pm} - B_{ij}^{pm} \cos \theta_{ij}^{pm}) & (j \neq i, m \neq p) \\ V_i^p B_{ii}^{pp} - Q_i^p / V_i^p & (j = i, m = p) \end{cases} \quad (5.62)$$

5.3 SSSC Modeling in Three-Phase Power Flow in Rectangular Coordinates

With the recent practical applications of converter based FACTS-devices such as the Static Synchronous Compensator (STATCOM) [26], Static Synchronous Series Compensator (SSSC) [27] and Unified Power Flow Controller (UPFC) [28], modeling and analysis of these FACTS-devices in power system operation and control is of great interest. In [24] mathematical models of the SSSC suitable for three-phase power flow analysis have been investigated.

In comparison to positive sequence model of SSSC, the three-phase SSSC models should consider:

- The differences between three-phase and positive sequence SSSC models. The three-phase SSSC models are basically different from the positive sequence SSSC models, which are able to give realistic results of power system operation with presence of unbalances of networks and loads while the positive sequence SSSC models can provide meaningful results only if both networks and loads are balanced.
- The transformer connection types. In the three-phase SSSC models, it is necessary to consider how the SSSC is connected with the transformer while, in the positive sequence SSSC models for conventional power flow calculations, such considerations are not needed.
- The similarity between three-phase models and positive sequence models. In principle, the three-phase models should be identical to the positive sequence models when both networks and loads are balanced.

5.3.1 Three-Phase SSSC Model with Delta/Wye Connected Transformer

5.3.1.1 Basic Operation Principles

Fig. 5.2 shows the basic operation principles of a three-phase SSSC. The SSSC consists of three converters, which are series connected with a three-phase transmission line via three single-phase transformers with Delta/Wye connections. The primary sides of the three single-phase transformers are delta-connected. It is assumed here that the transmission line is series connected with the SSSC bus j . With such an assumption, the active and reactive power flows entering the bus j are equal to the sending-end active and reactive power flows of the transmission line, respectively. In principle, the SSSC can generate and insert three-phase series voltage sources, which can be regulated to change the three-phase impedances (more precisely reactance) of the transmission line. In this way, the power of the transmission line, which the SSSC is connected with, can be controlled.

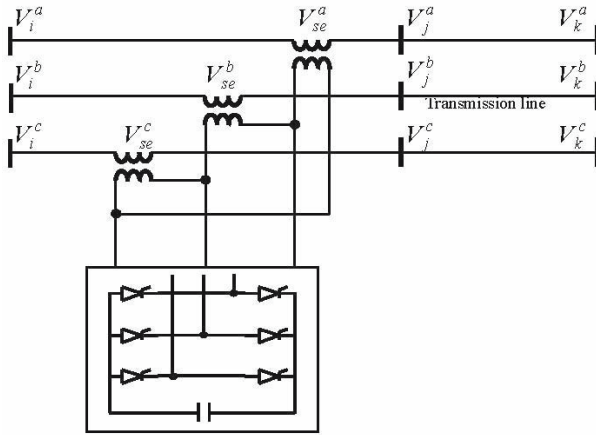


Fig. 5.2. Operating principles of three-phase SSSC with a Delta/Wye transformer

5.3.1.2 Equivalent Circuit of Three-Phase SSSC

The equivalent circuit of the three-phase SSSC is given in Fig. 5.3. The SSSC is represented by an ideal fundamental frequency three-phase voltage source vector V_{se}^{abc} in series with an impedance matrix Z_{se}^{abc} . Z_{se}^{abc} represents the impedance matrix of the three series transformers. The switching losses of SSSC may be included directly in Z_{se}^{abc} .

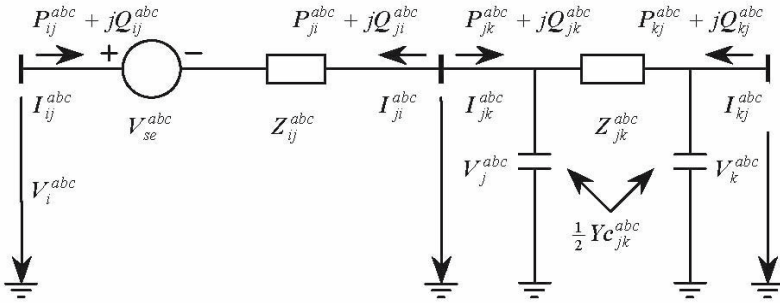


Fig. 5.3. Equivalent circuit of three-phase SSSC

$V_{se}^{abc} = [V_{se}^a, V_{se}^b, V_{se}^c]^T$ is the injected voltage vector. V_{se}^p ($p = a, b, c$) is the voltage phasor of phase p , which can be further represented by real and imaginary parts $V_{se}^p = E_{se}^p + jF_{se}^p$.

In the practical operation of the SSSC, the equivalent injected voltage magnitude of each phase should be within a specific voltage limit. We define:

$$0 \leq V_{se}^p \leq Vmax_{se}^p \quad (5.63)$$

where $p = a, b, c$. $V_{se}^p = \sqrt{(E_{se}^p)^2 + (F_{se}^p)^2}$. $Vmax_{se}^p$ is the voltage limit of phase p .

In the equivalent circuit, $V_i^{abc} = [V_i^a, V_i^b, V_i^c]^T$, $V_j^{abc} = [V_j^a, V_j^b, V_j^c]^T$, and $V_k^{abc} = [V_k^a, V_k^b, V_k^c]^T$ are the voltage vectors at buses i, j, k , respectively. $P_{ij}^{abc} = [P_{ij}^a, P_{ij}^b, P_{ij}^c]^T$ and $Q_{ij}^{abc} = [Q_{ij}^a, Q_{ij}^b, Q_{ij}^c]^T$ are active and reactive power flow vectors of branch ij leaving bus i . $P_{ji}^{abc} = [P_{ji}^a, P_{ji}^b, P_{ji}^c]^T$ and $Q_{ji}^{abc} = [Q_{ji}^a, Q_{ji}^b, Q_{ji}^c]^T$ are active and reactive power flow vectors of branch ij leaving bus j .

5.3.1.3 Power Equations of the Three-Phase SSSC

Assume that $V_m^p = E_m^p + jF_m^p$ ($m=i, j, k$ and $p=a, b, c$), the following power equations of the SSSC branch are derived according to the equivalent circuit shown in Fig. 5.3:

$$\begin{aligned}
P_{ij}^p = & E_i^p \sum_{m=a,b,c} (G_{ii}^{pm} E_i^m - B_{ii}^{pm} F_i^m) + F_i^p \sum_{m=a,b,c} (G_{ii}^{pm} F_i^m + B_{ii}^{pm} E_i^m) \\
& + E_i^p \sum_{m=a,b,c} (G_{ij}^{pm} E_j^m - B_{ij}^{pm} F_j^m) + F_i^p \sum_{m=a,b,c} (G_{ij}^{pm} F_j^m + B_{ij}^{pm} E_j^m) \\
& + E_i^p \sum_{m=a,b,c} (G_{ij}^{pm} E_{se}^m - B_{ij}^{pm} F_{se}^m) + F_i^p \sum_{m=a,b,c} (G_{ij}^{pm} F_{se}^m + B_{ij}^{pm} E_{se}^m)
\end{aligned} \quad (5.64)$$

$$\begin{aligned}
Q_{ij}^p = & -E_i^p \sum_{m=a,b,c} (G_{ii}^{pm} F_i^m + B_{ii}^{pm} E_i^m) + F_i^p \sum_{m=a,b,c} (G_{ii}^{pm} E_i^m - B_{ii}^{pm} F_i^m) \\
& - E_i^p \sum_{m=a,b,c} (G_{ij}^{pm} F_j^m + B_{ij}^{pm} E_j^m) + F_i^p \sum_{m=a,b,c} (G_{ij}^{pm} E_j^m - B_{ij}^{pm} F_j^m) \\
& - E_i^p \sum_{m=a,b,c} (G_{ij}^{pm} F_{se}^m + B_{ij}^{pm} E_{se}^m) + F_i^p \sum_{m=a,b,c} (G_{ij}^{pm} E_{se}^m - B_{ij}^{pm} F_{se}^m)
\end{aligned} \quad (5.65)$$

$$\begin{aligned}
P_{ji}^p = & E_j^p \sum_{m=a,b,c} (G_{jj}^{pm} E_j^m - B_{jj}^{pm} F_j^m) + F_j^p \sum_{m=a,b,c} (G_{jj}^{pm} F_j^m + B_{jj}^{pm} E_j^m) \\
& + E_j^p \sum_{m=a,b,c} (G_{ji}^{pm} E_i^m - B_{ji}^{pm} F_i^m) + F_j^p \sum_{m=a,b,c} (G_{ji}^{pm} F_i^m + B_{ji}^{pm} E_i^m) \\
& + E_j^p \sum_{m=a,b,c} (G_{ji}^{pm} E_{se}^m - B_{ji}^{pm} F_{se}^m) + F_j^p \sum_{m=a,b,c} (G_{ji}^{pm} F_{se}^m + B_{ji}^{pm} E_{se}^m)
\end{aligned} \quad (5.66)$$

$$\begin{aligned}
Q_{ji}^p = & -E_j^p \sum_{m=a,b,c} (G_{jj}^{pm} F_j^m + B_{jj}^{pm} E_j^m) + F_j^p \sum_{m=a,b,c} (G_{jj}^{pm} E_j^m - B_{jj}^{pm} F_j^m) \\
& - E_j^p \sum_{m=a,b,c} (G_{ji}^{pm} F_i^m + B_{ji}^{pm} E_i^m) + F_j^p \sum_{m=a,b,c} (G_{ji}^{pm} E_i^m - B_{ji}^{pm} F_i^m) \\
& - E_j^p \sum_{m=a,b,c} (G_{ji}^{pm} F_{se}^m + B_{ji}^{pm} E_{se}^m) + F_j^p \sum_{m=a,b,c} (G_{ji}^{pm} E_{se}^m - B_{ji}^{pm} F_{se}^m)
\end{aligned} \quad (5.67)$$

where $p = a, b, c$.

The power exchange of the three converters of the SSSC with the common DC link should be zero, which is as follows:

$$\Delta P_{se}^\Sigma = \sum_{p=a,b,c} P_{se}^p = 0 \quad (5.68)$$

where P_{se}^p is given by:

$$\begin{aligned}
P_{se}^p = & E_{se}^p \sum_{m=a,b,c} (G_{jj}^{pm} E_{se}^m - B_{jj}^{pm} F_{se}^m) + F_{se}^p \sum_{m=a,b,c} (G_{jj}^{pm} F_{se}^m + B_{jj}^{pm} E_{se}^m) \\
& + E_{se}^p \sum_{m=a,b,c} (G_{jj}^{pm} E_j^m - B_{jj}^{pm} F_j^m) + F_{se}^p \sum_{m=a,b,c} (G_{jj}^{pm} F_j^m + B_{jj}^{pm} E_j^m) \\
& + E_{se}^p \sum_{m=a,b,c} (G_{ji}^{pm} E_i^m - B_{ji}^{pm} F_i^m) + F_{se}^p \sum_{m=a,b,c} (G_{ji}^{pm} F_i^m + B_{ji}^{pm} E_i^m)
\end{aligned} \quad (5.69)$$

where $G_{ij}^{pm} + jB_{ij}^{pm} = G_{ji}^{pm} + jB_{ji}^{pm} = -y_{se}^{pm}$, $G_{ii}^{pm} + jB_{ii}^{pm} = y_{se}^{pm}$,

$G_{jj}^{pm} + jB_{jj}^{pm} = y_{se}^{pm}$ ($p = a, b, c$ and $m = a, b, c$). Here y_{se}^{pm} is given by:

$$\mathbf{Y}_{se}^{abc} = \begin{bmatrix} y_{se}^{aa} & y_{se}^{ab} & y_{se}^{ac} \\ y_{se}^{ba} & y_{se}^{bb} & y_{se}^{bc} \\ y_{se}^{ca} & y_{se}^{cb} & y_{se}^{cc} \end{bmatrix} = [\mathbf{Z}_{se}^{abc}]^{-1} = \begin{bmatrix} z_{se}^{aa} & 0 & 0 \\ 0 & z_{se}^{bb} & 0 \\ 0 & 0 & z_{se}^{cc} \end{bmatrix}^{-1} \quad (5.70)$$

where z_{se}^{aa} , z_{se}^{bb} and z_{se}^{cc} are the impedances of the three series transformers, respectively, in Fig. 5.2.

In the following, three models for the SSSC in Fig. 5.2 will be presented. They are the three-phase SSSC model with independent phase power control, three-phase SSSC model with total three-phase power control and three-phase SSSC model with symmetrical injected voltage control.

5.3.1.4 Three-Phase SSSC Model with Independent Phase Power Control

In the operation of the three-phase SSSC, the active power exchange of the three converters with the DC link should be zero. Such a constraint is described by (5.68). Besides, due to the fact that the SSSC is delta-connected with the three single-phase series transformers, the zero sequence component of the equivalent injected voltage vector $\mathbf{V}_{se}^{abc}$ should be zero. In other words, the following constraints should hold,

$$\Delta E_{se}^{\Sigma} = \text{Re}(\sum_{p=a,b,c} V_{se}^p) = \sum_{p=a,b,c} E_{se}^p = 0 \quad (5.71)$$

$$\Delta F_{se}^{\Sigma} = \text{Im}(\sum_{p=a,b,c} V_{se}^p) = \sum_{p=a,b,c} F_{se}^p = 0 \quad (5.72)$$

Since the SSSC steady model has six state variables such as $E_{se}^a, F_{se}^a, E_{se}^b, F_{se}^b, E_{se}^c, F_{se}^c$, it still has three control degrees of freedom. Here assuming that the three three-phase transmission line phase power flows can be controlled, we have:

$$\begin{aligned} \Delta P_{ji}^p &= P_{ji}^p - P_{spec_{ji}}^p = 0 \\ \Delta Q_{ji}^p &= Q_{ji}^p - Q_{spec_{ji}}^p = 0 \end{aligned} \quad (5.73)$$

where $p=a, b, c$. $P_{spec_{ji}}^p, Q_{spec_{ji}}^p$ are the control references of the active and reactive power flows, respectively, of phase p .

Combining the six operation and control constraint equations (5.68), (5.71)-(5.73) and six power mismatch equations at buses i, j together, the Newton power flow equation including the SSSC in rectangular coordinates may be given by:

$$\mathbf{J}\Delta\mathbf{X} = -\mathbf{F}(\mathbf{X}) \quad (5.74)$$

where

$$\begin{aligned} \Delta\mathbf{X} &= [\Delta\mathbf{X}_{sssc}, \Delta\mathbf{X}_{sys}]^T \\ \Delta\mathbf{X}_{sssc} &= [\Delta E_{se}^a, \Delta F_{se}^a, \Delta E_{se}^b, \Delta F_{se}^b, \Delta E_{se}^c, \Delta F_{se}^c]^T \\ \Delta\mathbf{X}_{sys} &= [\Delta E_i^a, \Delta F_i^a, \Delta E_i^b, \Delta F_i^b, \Delta E_i^c, \Delta F_i^c, \Delta E_j^a, \Delta F_j^a, \Delta E_j^b, \Delta F_j^b, \Delta E_j^c, \Delta F_j^c]^T \\ \mathbf{F}(\mathbf{X}) &= [\mathbf{F}_{sssc}, \mathbf{F}_{sys}]^T \\ \mathbf{F}_{sssc} &= [\Delta P_{se}^\Sigma, \Delta E_{se}^\Sigma, \Delta F_{se}^\Sigma, \Delta P_{ji}^a, \Delta P_{ji}^b, \Delta P_{ji}^c]^T \\ \mathbf{F}_{sys} &= [\Delta P_i^a, \Delta Q_i^a, \Delta P_i^b, \Delta Q_i^b, \Delta P_i^c, \Delta Q_i^c, \Delta P_j^a, \Delta Q_j^a, \Delta P_j^b, \Delta Q_j^b, \Delta P_j^c, \Delta Q_j^c]^T \\ \mathbf{J} &= \frac{\partial \mathbf{F}(\mathbf{X})}{\partial \mathbf{X}} \end{aligned}$$

5.3.1.5 Three-Phase SSSC Model with Total Three-Phase Power Control

Assume, for the SSSC in Fig. 5.2, that (a) the three equivalent injected voltages $V_{se}^a, V_{se}^b, V_{se}^c$ are perpendicular to the line currents of phase a, phase b, and phase c, respectively; (b) the total three-phase power is controlled, then:

$$P_{se}^p = 0 \quad (5.75)$$

and

$$\begin{aligned} \Delta P_{ji}^\Sigma &= \sum_{p=a,b,c} P_{ji}^p - P_{spec}^\Sigma = 0 \\ \text{or } \Delta Q_{ji}^\Sigma &= \sum_{p=a,b,c} Q_{ji}^p - Q_{spec}^\Sigma = 0 \end{aligned} \quad (5.76)$$

where $p = a, b, c$. P_{spec}^Σ and Q_{spec}^Σ are the specified total three-phase active and reactive power flow control references, respectively.

Combining the six operation and control constraint equations (5.71), (5.72), (5.75) and (5.76) and six power mismatch equations at buses i, j together, the Newton power flow equation including the SSSC in rectangular coordinates may be given by:

$$\mathbf{J}\Delta\mathbf{X} = -\mathbf{F}(\mathbf{X}) \quad (5.77)$$

where

$$\Delta\mathbf{X} = [\Delta\mathbf{X}_{sssc}, \Delta\mathbf{X}_{sys}]^T$$

$$\begin{aligned}
\Delta \mathbf{X}_{sssc} &= [\Delta E_{se}^a, \Delta F_{se}^a, \Delta E_{se}^b, \Delta F_{se}^b, \Delta E_{se}^c, \Delta F_{se}^c]^T \\
\Delta \mathbf{X}_{sys} &= [\Delta E_i^a, \Delta F_i^a, \Delta E_i^b, \Delta F_i^b, \Delta E_i^c, \Delta F_i^c, \Delta E_j^a, \Delta F_j^a, \Delta E_j^b, \Delta F_j^b, \Delta E_j^c, \Delta F_j^c]^T \\
\mathbf{F}(\mathbf{X}) &= [\mathbf{F}_{sssc}, \mathbf{F}_{sys}]^T \\
\mathbf{F}_{sssc} &= [P_{se}^a, P_{se}^b, P_{se}^c, \Delta E_{se}^\Sigma, \Delta F_{se}^\Sigma, \Delta P_{ji}^\Sigma]^T \\
\mathbf{F}_{sys} &= [\Delta P_i^a, \Delta Q_i^a, \Delta P_i^b, \Delta Q_i^b, \Delta P_i^c, \Delta Q_i^c, \Delta P_j^a, \Delta Q_j^a, \Delta P_j^b, \Delta Q_j^b, \Delta P_j^c, \Delta Q_j^c]^T \\
\mathbf{J} &= \frac{\partial \mathbf{F}(\mathbf{X})}{\partial \mathbf{X}}
\end{aligned}$$

5.3.1.6 Three-Phase SSSC Model with Symmetrical Injected Voltage Control

If we assume the series injected three-phase voltage sources of the three-phase SSSC are balanced or symmetrical, then we have the following control constraint equations:

$$\mathbf{V}_{se}^a = \mathbf{V}_{se}^b e^{j120^\circ} = \mathbf{V}_{se}^c e^{j240^\circ} \quad (5.78)$$

A set of symmetrical or balanced three-phase voltage phasors are equal in magnitude while their phase angles have 120° displacement among them. For the sake of computation, equation (5.78) may be replaced by the following four equations in real and imaginary parts:

$$\Delta V_{Re}^1 = \text{Re}(\mathbf{V}_{se}^a - \mathbf{V}_{se}^b e^{j120^\circ}) = E_{se}^a + \frac{1}{2} E_{se}^b + \frac{\sqrt{3}}{2} F_{se}^b = 0 \quad (5.79)$$

$$\Delta V_{Im}^1 = \text{Im}(\mathbf{V}_{se}^a - \mathbf{V}_{se}^b e^{j120^\circ}) = E_{se}^a - \frac{\sqrt{3}}{2} E_{se}^b + \frac{1}{2} F_{se}^b = 0 \quad (5.80)$$

$$\Delta V_{Re}^2 = \text{Re}(\mathbf{V}_{se}^a - \mathbf{V}_{se}^c e^{j240^\circ}) = E_{se}^a + \frac{1}{2} E_{se}^c - \frac{\sqrt{3}}{2} F_{se}^c = 0 \quad (5.81)$$

$$\Delta V_{Im}^2 = \text{Im}(\mathbf{V}_{se}^a - \mathbf{V}_{se}^c e^{j240^\circ}) = F_{se}^a + \frac{\sqrt{3}}{2} E_{se}^c + \frac{1}{2} F_{se}^c = 0 \quad (5.82)$$

When (5.79)-(5.82) hold, equations (5.71) and (5.72) will be satisfied. Furthermore, the active power exchange constraint (5.68) should be balanced at any instant. So the SSSC with symmetrical control has only one control degree of freedom. Assuming that the total active power or reactive power of the three-phase transmission line is controlled, we have the following power flow control constraint:

$$\begin{aligned}
\Delta P_{ji}^\Sigma &= \sum_{p=a,b,c} P_{ji}^p - P_{spec}^\Sigma = 0 \\
\text{or } \Delta Q_{ji}^\Sigma &= \sum_{p=a,b,c} Q_{ji}^p - Q_{spec}^\Sigma = 0
\end{aligned} \quad (5.83)$$

where $p = a, b, c$. $PSpec_{ji}^{\Sigma}$ and $QSpec_{ji}^{\Sigma}$ are the specified total three-phase active and reactive power flow control references, respectively.

The SSSC model with symmetrical voltage control has six operation and control constraint equations (5.68), (5.79)–(5.82). Combining the six operation and control constraint equations (5.68), (5.79)–(5.82) and six power mismatch equations at buses i, j together, the Newton power flow equation including the SSSC in rectangular coordinates may be given by:

$$\mathbf{J}\Delta\mathbf{X} = -\mathbf{F}(\mathbf{X}) \quad (5.84)$$

where $\Delta\mathbf{X} = [\Delta\mathbf{X}_{sssc}, \Delta\mathbf{X}_{sys}]^T$

$$\Delta\mathbf{X}_{sssc} = [\Delta E_{se}^a, \Delta F_{se}^a, \Delta E_{se}^b, \Delta F_{se}^b, \Delta E_{se}^c, \Delta F_{se}^c]^T$$

$$\Delta\mathbf{X}_{sys} = [\Delta E_i^a, \Delta F_i^a, \Delta E_i^b, \Delta F_i^b, \Delta E_i^c, \Delta F_i^c, \Delta E_j^a, \Delta F_j^a, \Delta E_j^b, \Delta F_j^b, \Delta E_j^c, \Delta F_j^c]^T$$

$$\mathbf{F}(\mathbf{X}) = [\mathbf{F}_{sssc}, \mathbf{F}_{sys}]^T$$

$$\mathbf{F}_{sssc} = [\Delta P_{se}^{\Sigma}, \Delta V_{Re}^1, \Delta V_{Im}^1, \Delta V_{Re}^2, \Delta V_{Im}^2, \Delta P_{ji}^{\Sigma}]^T$$

$$\mathbf{F}_{sys} = [\Delta P_i^a, \Delta Q_i^a, \Delta P_i^b, \Delta Q_i^b, \Delta P_i^c, \Delta Q_i^c, \Delta P_j^a, \Delta Q_j^a, \Delta P_j^b, \Delta Q_j^b, \Delta P_j^c, \Delta Q_j^c]^T$$

$$\mathbf{J} = \frac{\partial \mathbf{F}(\mathbf{X})}{\partial \mathbf{X}}$$

5.3.2 Single-Phase/Three-Phase SSSC Models with Separate Single Phase Transformers

5.3.2.1 Basic Operating Principles of Single Phase SSSC

In Fig 5.4, three single-phase SSSCs are series connected with phase a, b, c of a transmission line, respectively. The three SSSCs have neither electrical nor magnetic connections between them. Each single phase SSSC is series-connected with the transmission line via a single-phase transformer. Each can independently control the phase power flow of the transmission line. The single-phase SSSC is attractive and practical when there are unbalanced loads and one or two phase lines existing in the systems.

5.3.2.2 Equivalent Circuit of Single Phase SSSC

Due to the fact that there are no electrical and magnetic couplings between the three single-phase SSSCs, each SSSC branch can be represented by an equivalent circuit shown in Fig. 5.5. Such an equivalent circuit is exactly the same to that of the SSSC for the positive sequence power flow calculations. However, the physical meaning of the single-phase equivalent circuit here is quite different from that of the positive sequence SSSC in the positive sequence power flow calculations.

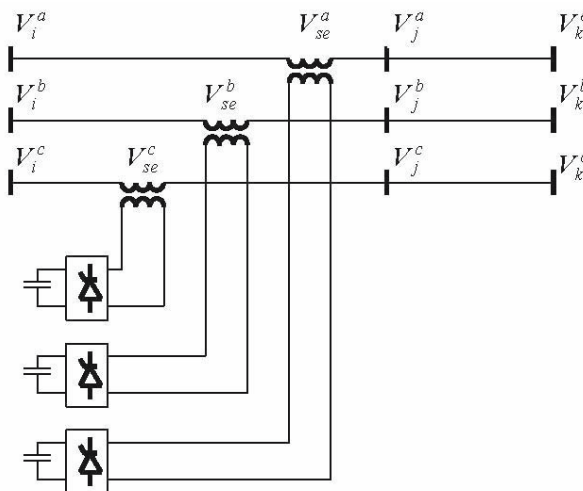


Fig. 5.4. Operation principle of single phase SSSC for three-phase power flow analysis

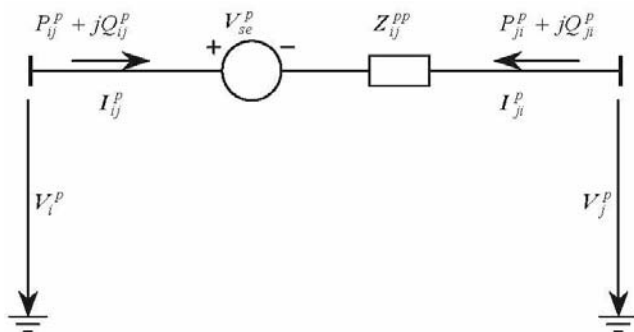


Fig. 5.5. Equivalent circuit of single phase SSSC for three-phase power flow analysis

5.3.2.3 Single-Phase SSSC

The power flow equations (5.64)-(5.67) for the three-phase SSSC are still applicable to the system with the three separate single-phase SSSCs installed on phase a , b , c of the transmission line in Fig. 5.4, respectively.

Since each single phase SSSC can neither generate nor absorb active power, the power exchange of each SSSC with the system should be zero. Mathematically, such a constraint may be represented by:

$$P_{se}^p = 0 \quad (5.85)$$

where $p=a, b, c$. P_{se}^p , which is given by (5.69), is the active power exchange of SSSC with the DC link or the system.

Assuming that each SSSC independently controls the phase active or reactive power flow of the transmission line, the power flow control constraint may be represented by:

$$P_{ji}^p - P_{spec}^p_{ji} = 0 \quad \text{or} \quad Q_{ji}^p - Q_{spec}^p_{ji} = 0 \quad (5.86)$$

where $p=a, b, c$. $P_{spec}^p_{ji}$ and $Q_{spec}^p_{ji}$ are the specified active and reactive power flow control references of phase p , respectively.

Combining the six operation and control equations (5.85) and (5.86) of the three single phase SSSCs and six power mismatch equations of buses i and j , the three-phase Newton equation may be given by:

$$\mathbf{J}\Delta\mathbf{X} = -\mathbf{F}(\mathbf{X}) \quad (5.87)$$

where $\Delta\mathbf{X} = [\Delta\mathbf{X}_{sssc}, \Delta\mathbf{X}_{sys}]^T$

$$\Delta\mathbf{X}_{sssc} = [\Delta E_{se}^a, \Delta F_{se}^a, \Delta E_{se}^b, \Delta F_{se}^b, \Delta E_{se}^c, \Delta F_{se}^c]^T$$

$$\Delta\mathbf{X}_{sys} = [\Delta E_i^a, \Delta F_i^a, \Delta E_i^b, \Delta F_i^b, \Delta E_i^c, \Delta F_i^c, \Delta E_j^a, \Delta F_j^a, \Delta E_j^b, \Delta F_j^b, \Delta E_j^c, \Delta F_j^c]^T$$

$$\mathbf{F}(\mathbf{X}) = [\mathbf{F}_{sssc}, \mathbf{F}_{sys}]^T$$

$$\mathbf{F}_{sssc} = [P_{se}^a, P_{se}^b, P_{se}^c, \Delta P_{ji}^a, \Delta P_{ji}^b, \Delta P_{ji}^c]^T$$

$$\mathbf{F}_{sys} = [\Delta P_i^a, \Delta Q_i^a, \Delta P_i^b, \Delta Q_i^b, \Delta P_i^c, \Delta Q_i^c, \Delta P_j^a, \Delta Q_j^a, \Delta P_j^b, \Delta Q_j^b, \Delta P_j^c, \Delta Q_j^c]^T$$

$$\mathbf{J} = \frac{\partial \mathbf{F}(\mathbf{X})}{\partial \mathbf{X}}$$

5.3.2.4 Three-Phase SSSC Model with Three Separate Single Phase Transformers

If we assume (a) a three-phase SSSC is connected with a three-phase transmission line via three separate single phase transformers; (b) the three injected voltages $\mathbf{V}_{se}^a, \mathbf{V}_{se}^b, \mathbf{V}_{se}^c$ of the SSSC are perpendicular to the line currents of phase a, phase b, and phase c of the transmission line, respectively; (c) the three single phase power flows can be controlled, then the three-phase SSSC will have similar constraint equations of (5.85) and (5.86). Subsequently, for the three-phase SSSC, we have the similar Newton equation as given by (5.87).

5.3.3 Numerical Examples

A 5-bus system and the IEEE 118 bus system have been used to test the three-phase Newton power flow algorithm with modeling of the SSSC. The 5 bus three-phase system is shown in Fig. 5.8 in the Appendix of this chapter while the system parameters are listed in Table 5.11 - Table 5.14. In the tests, a convergence tolerance of $1.0\text{e-}12$ p.u. is used. For the sake of convenience, the three-phase SSSC model with independent phase power control, three-phase SSSC model with total three-phase power control and three-phase SSSC model with symmetrical injected voltage control in Section 2 are denoted as Model 1, Model 2 and Model 3, respectively, while the three-phase and single phase SSSC models in Section 3 are referred to Model 4 and Model 5, respectively.

5.3.3.1 Test Results for the 5-Bus System

Based on the 5-bus system, tests under the following conditions have been carried out:

- Case 1:* Well transposed transmission lines and the whole system with balanced load.
- Case 2:* Non-transposed transmission lines and unbalanced load at bus 3 as given by Table 5.13 and Table 5.14.
- Case 3:* As for case 1, but a SSSC is installed at the sending-end of the transmission line 1-3.
- Case 4:* As for case 2, but a SSSC is installed at the sending-end of the transmission line 1-3.
- Case 5:* As for case 3, but the whole system is represented by the positive sequence network only.

The number of iterations of the three-phase power flow algorithm on cases 1-4 are summarized in Table 5.1. For cases 3 and 4, the control references of the SSSC Models 1, 4 and 5 are $Pspec_{ji}^a = Pspec_{ji}^b = Pspec_{ji}^c = 7.0\text{p.u.}$ while the control reference for the SSSC Models 2 and 3 is $Pspec_{ji}^{\Sigma} = 21.0\text{p.u.}$ In order to verify the validity of the three-phase power flow algorithm and the SSSC models, case 5 has been carried out, in which the whole system is represented only by the positive sequence network since the system is balanced. The power flow solution of case 5 is obtained by a positive sequence power flow program. The detailed power flow solutions of case 3 and case 5 are given by Table 5.2.

Table 5.1. Number of iterations of three-phase power flow algorithm for the 5-bus system

Case No.	Base case power flows / SSSC power flow solutions	Total power flow increase (%)	Number of iterations
1	$P_{13}^a = P_{13}^b = P_{13}^c = 4.94 p.u.$ $Q_{13}^a = Q_{13}^b = Q_{13}^c = 2.01 p.u.$ $P_{13}^\Sigma = 14.82 p.u.$ $Q_{13}^\Sigma = 6.03 p.u.$	-	6
2	$P_{13}^a = 4.96 p.u.$ $P_{13}^b = 5.17 p.u.$ $P_{13}^c = 4.69 p.u.$ $Q_{13}^a = 1.96 p.u.$ $Q_{13}^b = 1.46 p.u.$ $Q_{13}^c = 2.33 p.u.$ $P_{13}^\Sigma = 14.82 p.u.$ $Q_{13}^\Sigma = 5.75 p.u.$	-	6
3	SSSC Model 1: $V_{se}^a = V_{se}^b = V_{se}^c = 0.1933 p.u.$ $\theta_{se}^a = 283.83^\circ$ $\theta_{se}^b = 163.83^\circ$ $\theta_{se}^c = 43.83^\circ$	42%	6
	SSSC Model 2: $V_{se}^a = V_{se}^b = V_{se}^c = 0.1933 p.u.$ $\theta_{se}^a = 283.83^\circ$ $\theta_{se}^b = 163.83^\circ$ $\theta_{se}^c = 43.83^\circ$	42%	6
	SSSC Model 3: $V_{se}^a = V_{se}^b = V_{se}^c = 0.1933 p.u.$ $\theta_{se}^a = 283.83^\circ$ $\theta_{se}^b = 163.83^\circ$ $\theta_{se}^c = 43.83^\circ$	42%	6
	SSSC Models 4 and 5: $V_{se}^a = V_{se}^b = V_{se}^c = 0.1933 p.u.$ $\theta_{se}^a = 283.83^\circ$ $\theta_{se}^b = 163.83^\circ$ $\theta_{se}^c = 43.83^\circ$	42%	6
4	SSSC Model 1: $V_{se}^a = 0.1781 p.u.$ $V_{se}^b = 0.1400 p.u.$ $V_{se}^c = 0.2301 p.u.$ $\theta_{se}^a = 265.68^\circ$ $\theta_{se}^b = 177.55^\circ$ $\theta_{se}^c = 48.22^\circ$	42%	6
	SSSC Model 2: $V_{se}^a = 0.1437 p.u.$ $V_{se}^b = 0.1836 p.u.$ $V_{se}^c = 0.1663 p.u.$ $\theta_{se}^a = 288.49^\circ$ $\theta_{se}^b = 168.08^\circ$ $\theta_{se}^c = 36.25^\circ$	42%	6
	SSSC Model 3: $V_{se}^a = V_{se}^b = V_{se}^c = 0.1661 p.u.$ $\theta_{se}^a = 283.96^\circ$ $\theta_{se}^b = 163.95^\circ$ $\theta_{se}^c = 43.95^\circ$	42%	6
	SSSC Models 4 and 5: $V_{se}^a = 0.1730 p.u.$ $V_{se}^b = 0.1100 p.u.$ $V_{se}^c = 0.2443 p.u.$ $\theta_{se}^a = 284.64^\circ$ $\theta_{se}^b = 165.36^\circ$ $\theta_{se}^c = 41.53^\circ$	42%	6

Table 5.2. Power flow solutions for the 5 bus system by three-phase and single-phase power flow algorithms

Case 3			Case 5		
Bus No.	V^a (p.u.)	θ^a (deg.)	Bus No.	V (p.u.)	θ (deg.)
1	1.0183	27.35	1	1.0183	-2.65
2	1.0238	28.64	2	1.0238	-1.36
3	1.0101	30.47	3	1.0101	0.47
4	1.0450	0.00	4	1.0450	0.00
5	1.0610	2.70	5	1.0610	2.70
SSSC	$V_{se}^a = 0.1933$	$\theta_{se}^a = 283.83$	SSSC	$V_{se} = 0.1933$	$\theta_{se} = 253.83$

From Table 5.1 and Table 5.2, it can be seen,

1. The three-phase power flow algorithm with incorporation of the SSSC models can converge in only 6 iterations.
2. In case 3, the power flow solutions with the different SSSC models are the same when the system is balanced. Such a coincidence of computation results implies the validity of the SSSC models proposed.
3. Comparison of power flow solutions of case 3 and case 5 in Table 5.2 indicates that two sets of solutions by the conventional positive sequence power flow algorithm and three-phase power flow algorithm proposed are almost identical. The only difference is that there is 30 degree angle shift in the three-phase power flow results which is caused by the Delta/Wye transformers. The comparison of the two power flow solutions again illustrates the validity of the three-phase power flow algorithm and the SSSC models. It can be anticipated that if Wye/Wye transformers rather than Delta/Wye transformers are used, the power flow solutions by the conventional positive sequence power flow and the three-phase power flow computations should be the same, and the 30 degree shift should disappear. This theoretical analysis has been confirmed by power flow calculations. Due to the limitation of space, the power flow calculation results are not presented herein.
4. Case 4 in Table 5.1 shows that the different SSSC models will have different power flow solutions when the system is unbalanced. This implies that the appropriate modeling of the SSSC in three-phase power flow calculations is very important.

5.3.3.2 Test Results for the IEEE 118-Bus System

Further tests have been carried out on the IEEE 118-bus system, which are as follows:

Case 6: Well-transposed transmission lines and the whole system with balanced load.

- Case 7:* Well-transposed transmission lines and the system with unbalanced load at bus 78 with $0.51+j0.26$ p.u., $0.71+j0.26$ p.u., $0.91+j0.26$ p.u. for phase a, b, c loading, respectively.
- Case 8:* As for case 6, there are two SSSCs installed on the transmission lines 30-38 and 68-81.
- Case 9:* As for case 7, there are two SSSCs installed on the transmission lines 30-38 and 68-81.

In cases 8 and 9, the control references of the SSSCs are 140% of the base case power flows, respectively. The test results are given by Table 5.3. The convergence characteristics of case 7 and case 9 are shown in Fig. 5.6, from which it can be seen that the power flow algorithm exhibits excellent quadratic convergence characteristics.

Table 5.3. Test results on the IEEE 118-bus system

Case No.	SSSC models	Number of iterations
6	None	6
7	None	6
8	The SSSC on line 30-38: Model 1	6
	The SSSC on line 68-81: Model 3	6
9	The SSSC on line 30-38: Model 3	6
	The SSSC on line 68-81: Model 4	6

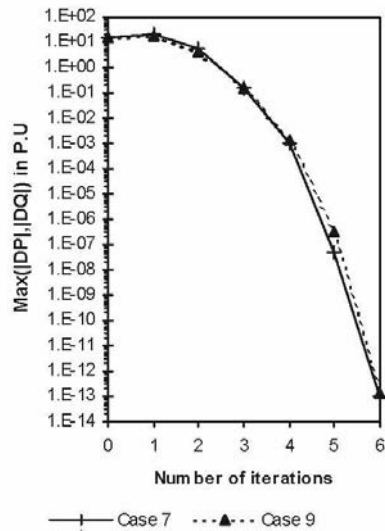


Fig. 5.6. Power mismatches as function of number of iterations on the IEEE 118-bus three-phase system

5.4 UPFC Modeling in Three-Phase Newton Power Flow in Polar Coordinates

In section 5.3, the mathematical models of SSSC for three-phase power flow analysis have been proposed. The UPFC combining two three-phase converters has been considered as one of the most powerful converter based FACTS-devices and can be used to control power flows and bus voltages. It has been recognized that due to the relative simplicity of the SSSC transformers, the transformer connection types are just implicitly represented.

However, due to the complicated combinations of the converter topologies and transformer connection types, for the modeling of the UPFC in three-phase power flow analysis, the implicit representation of the converter transformers cannot be considered adequate and may have modeling limitations. Hence, the representation of transformer connection types and UPFC control constraints becomes essential [25]. In this section, the mathematical models of UPFC, in polar coordinates, for three-phase power flow analysis are discussed. With the UPFC models derived, three-phase STATCOM models can be easily derived by eliminating the series part constraints from the equations.

5.4.1 Operation Principles of the Three-Phase UPFC

Fig. 5.7 shows the basic operating principles of a three-phase UPFC. The UPFC consists of series converter and shunt converter. The series converter is series connected with a three-phase transmission line via three single-phase transformers. The shunt-converter is coupled with the ac bus via a three-phase Wye-G/Delta transformer. It should be pointed out that Fig. 5.7 is just used to show one of the topologies and the related transformer connection types of the three-phase UPFC. However, in addition to the Wye-G/Delta connection, the UPFC may have other kinds of series and shunt transformer connection types, which will be considered in the following derivation.

In Fig. 5.7, it is assumed that the transmission line is series connected with the UPFC bus j . With such an assumption, the active and reactive power flows entering bus j are equal to the sending-end active and reactive power flows of the transmission line, respectively. In principle, the series converter may be used to control the active and reactive power flows of the transmission line while the shunt converter can be used to control the voltages of the shunt bus.

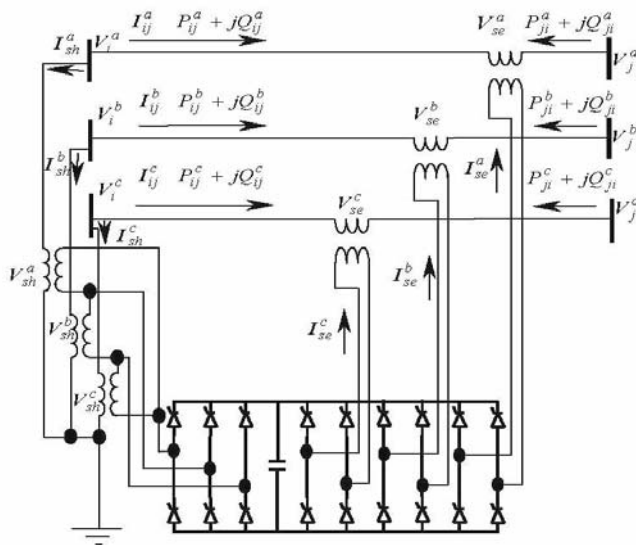


Fig. 5.7. Schematic description of a three-phase UPFC

5.4.2 Three-Phase Converter Transformer Models

In single-phase positive-sequence power flow analysis, it is usually sufficient to represent a three-phase transformer as a positive-sequence impedance in series with an ideal transformer. However, in three-phase power flow analysis where unbalanced operating conditions of network and load are considered, such a transformer model can no longer be considered appropriate. In principle, the three-phase transformer should be described in three-phase coordinates and the connection type of the transformer should also be fully represented. The admittance transformer models of various connections in three-phase coordinates were derived [23]. A two winding three-phase transformer may be represented by:

$$\begin{bmatrix} \mathbf{I}_P^{abc} \\ \mathbf{I}_S^{abc} \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{PP}^{abc} & \mathbf{Y}_{PS}^{abc} \\ \mathbf{Y}_{SP}^{abc} & \mathbf{Y}_{SS}^{abc} \end{bmatrix} \begin{bmatrix} \mathbf{V}_P^{abc} \\ \mathbf{V}_S^{abc} \end{bmatrix} \quad (5.88)$$

where $\mathbf{I}_P^{abc}$ and $\mathbf{I}_S^{abc}$ are the current vectors of the primary and secondary windings, respectively while $\mathbf{V}_P^{abc}$ and $\mathbf{V}_S^{abc}$ are the voltage vectors of the primary and secondary windings, respectively. $\mathbf{Y}_{PP}^{abc}$, $\mathbf{Y}_{SS}^{abc}$, $\mathbf{Y}_{PS}^{abc}$ and $\mathbf{Y}_{SP}^{abc}$ are 3 by 3 submatrices and they are given by Table 5.4 based on transformer connection types.

Table 5.4. Submatrices of three-phase transformers with different connection types

Type No.	Transformer connection		Self admittance		Mutual admittance	
	Bus P	Bus S	$\mathbf{Y}_{PP}^{abc}$	$\mathbf{Y}_{SS}^{abc}$	$\mathbf{Y}_{PS}^{abc}$	$\mathbf{Y}_{SP}^{abc}$
1	Wye-G	Wye-G	$\mathbf{Y}_I$	$\mathbf{Y}_I$	$-\mathbf{Y}_I$	$-\mathbf{Y}_I$
2	Wye-G	Wye	$\mathbf{Y}_{II}$	$\mathbf{Y}_{II}$	$-\mathbf{Y}_{II}$	$-\mathbf{Y}_{II}$
3	Wye-G	Delta	$\mathbf{Y}_I$	$\mathbf{Y}_{II}$	$\mathbf{Y}_{III}$	$\mathbf{Y}_{III}^T$
4	Wye	Wye	$\mathbf{Y}_{II}$	$\mathbf{Y}_{II}$	$-\mathbf{Y}_{II}$	$-\mathbf{Y}_{II}$
5	Wye	Delta	$\mathbf{Y}_{II}$	$\mathbf{Y}_{II}$	$\mathbf{Y}_{III}$	$\mathbf{Y}_{III}^T$
6	Delta	Delta	$\mathbf{Y}_{II}$	$\mathbf{Y}_{II}$	$-\mathbf{Y}_{II}$	$-\mathbf{Y}_{II}$
7	Wye	Wye-G	$\mathbf{Y}_{II}$	$\mathbf{Y}_{II}$	$-\mathbf{Y}_{II}$	$-\mathbf{Y}_{II}$
8	Delta	Wye-G	$\mathbf{Y}_{II}$	$\mathbf{Y}_I$	$\mathbf{Y}_{III}^T$	$\mathbf{Y}_{III}$
9	Delta	Wye	$\mathbf{Y}_{II}$	$\mathbf{Y}_{II}$	$\mathbf{Y}_{III}^T$	$\mathbf{Y}_{III}$

In Table 5.4, $\mathbf{Y}_I$, $\mathbf{Y}_{II}$ and $\mathbf{Y}_{III}$ are defined by:

$$\mathbf{Y}_I = \begin{bmatrix} y_t & 0 & 0 \\ 0 & y_t & 0 \\ 0 & 0 & y_t \end{bmatrix} \quad (5.89)$$

$$\mathbf{Y}_{II} = \frac{1}{3} \begin{bmatrix} 2y_t & -y_t & -y_t \\ -y_t & 2y_t & -y_t \\ -y_t & -y_t & 2y_t \end{bmatrix} \quad (5.90)$$

$$\mathbf{Y}_{III} = \frac{1}{\sqrt{3}} \begin{bmatrix} -y_t & y_t & 0 \\ 0 & -y_t & y_t \\ y_t & 0 & -y_t \end{bmatrix} \quad (5.91)$$

where y_t is the per unit leakage admittance.

If the transformer has off-nominal tap ratios α and β of the primary and secondary windings, respectively, then the self and mutual matrices need to be modified by:

1. dividing the primary self admittance matrix $\mathbf{Y}_{PP}^{abc}$ by α^2 ;
2. dividing the secondary self admittance matrix $\mathbf{Y}_{SS}^{abc}$ by β^2 ;
3. dividing the mutual admittance matrices $\mathbf{Y}_{PS}^{abc}$ and $\mathbf{Y}_{SP}^{abc}$ by $\alpha\beta$.

5.4.3 Power Flow Constraints of the Three-Phase UPFC

5.4.3.1 Power Flow Constraints of the Shunt Converter

Based on the operating principles shown in Fig. 5.6, the bus voltage equation of the three-phase shunt converter transformer of the UPFC may be given by:

$$\begin{bmatrix} \mathbf{I}_{ii}^{abc} \\ \mathbf{I}_{sh}^{abc} \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{sh_{PP}}^{abc} & \mathbf{Y}_{sh_{PS}}^{abc} \\ \mathbf{Y}_{sh_{SP}}^{abc} & \mathbf{Y}_{sh_{SS}}^{abc} \end{bmatrix} \begin{bmatrix} \mathbf{V}_i^{abc} \\ \mathbf{V}_{sh}^{abc} \end{bmatrix} \quad (5.92)$$

where $\mathbf{I}_{ii}^{abc}$ and $\mathbf{I}_{sh}^{abc}$ are the current vectors of the primary and secondary windings of the shunt converter transformer and given by:

$$\mathbf{I}_{ii}^{abc} = [\mathbf{I}_{ii}^a, \mathbf{I}_{ii}^b, \mathbf{I}_{ii}^c]^T \quad (5.93)$$

$$\mathbf{I}_{sh}^{abc} = [\mathbf{I}_{sh}^a, \mathbf{I}_{sh}^b, \mathbf{I}_{sh}^c]^T \quad (5.94)$$

The voltage vectors are:

$$\mathbf{V}_i^{abc} = [\mathbf{V}_i^a, \mathbf{V}_i^b, \mathbf{V}_i^c]^T \quad (5.95)$$

$$\mathbf{V}_{sh}^{abc} = [\mathbf{V}_{sh}^a, \mathbf{V}_{sh}^b, \mathbf{V}_{sh}^c]^T \quad (5.96)$$

The active and reactive power flows of the primary side of the shunt converter transformer are:

$$\begin{aligned} P_{ii}^p &= \text{Re}[\mathbf{V}_i^p (\mathbf{I}_{ii}^p)^*] \\ &= \sum_{m=a,b,c} V_i^p V_i^m (G_{sh_{PP}}^{pm} \cos(\theta_i^p - \theta_i^m) + B_{sh_{PP}}^{pm} \sin(\theta_i^p - \theta_i^m)) \end{aligned} \quad (5.97)$$

$$+ \sum_{m=a,b,c} V_i^p V_{sh}^m (G_{sh_{PS}}^{pm} \cos(\theta_i^p - \theta_{sh}^m) + B_{sh_{PS}}^{pm} \sin(\theta_i^p - \theta_{sh}^m))$$

$$\begin{aligned} Q_{ii}^p &= \text{Im}[\mathbf{V}_i^p (\mathbf{I}_{ii}^p)^*] \\ &= \sum_{m=a,b,c} V_i^p V_i^m (G_{sh_{PP}}^{pm} \sin(\theta_i^p - \theta_i^m) - B_{sh_{PP}}^{pm} \cos(\theta_i^p - \theta_i^m)) \end{aligned} \quad (5.98)$$

$$+ \sum_{m=a,b,c} V_i^p V_{sh}^m (G_{sh_{PS}}^{pm} \sin(\theta_i^p - \theta_{sh}^m) - B_{sh_{PS}}^{pm} \cos(\theta_i^p - \theta_{sh}^m))$$

where $p, m = a, b, c$. $G_{sh_{PP}}^{pm} + jB_{sh_{PP}}^{pm} \in \mathbf{Y}_{sh_{PP}}^{abc}$, and $G_{sh_{PS}}^{pm} + jB_{sh_{PS}}^{pm} \in \mathbf{Y}_{sh_{PS}}^{abc}$.

Similarly, the active and reactive power flows at the secondary side of the shunt converter transformer are given by:

$$\begin{aligned}
P_{sh}^p &= \text{Re}[V_{sh}^p (\mathbf{I}_{sh}^p)^*] \\
&= \sum_{m=a,b,c} V_{sh}^p V_{sh}^m (Gsh_{SS}^{pm} \cos(\theta_{sh}^p - \theta_{sh}^m) + Bsh_{SS}^{pm} \sin(\theta_{sh}^p - \theta_{sh}^m)) \\
&\quad + \sum_{m=a,b,c} V_{sh}^p V_i^m (Gsh_{SP}^{pm} \cos(\theta_{sh}^p - \theta_i^m) + Bsh_{SP}^{pm} \sin(\theta_{sh}^p - \theta_i^m))
\end{aligned} \tag{5.99}$$

$$\begin{aligned}
Q_{sh}^p &= \text{Im}[V_{sh}^p (\mathbf{I}_{sh}^p)^*] \\
&= \sum_{m=a,b,c} V_{sh}^p V_{sh}^m (Gsh_{SS}^{pm} \sin(\theta_{sh}^p - \theta_{sh}^m) - Bsh_{SS}^{pm} \cos(\theta_{sh}^p - \theta_{sh}^m)) \\
&\quad + \sum_{m=a,b,c} V_{sh}^p V_i^m (Gsh_{SP}^{pm} \sin(\theta_{sh}^p - \theta_i^m) - Bsh_{SP}^{pm} \cos(\theta_{sh}^p - \theta_i^m))
\end{aligned} \tag{5.100}$$

5.4.3.2 Power Flow Constraints of the Series Converter

Based on the operating principles shown in Fig. 5.7, the bus voltage equation of the series converter transformer may be generally given by:

$$\begin{bmatrix} \mathbf{I}_{ij}^{abc} \\ \mathbf{I}_{se}^{abc} \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{sePP}^{abc} & \mathbf{Y}_{sePS}^{abc} \\ \mathbf{Y}_{seSP}^{abc} & \mathbf{Y}_{seSS}^{abc} \end{bmatrix} \begin{bmatrix} \mathbf{V}_i^{abc} - \mathbf{V}_j^{abc} \\ \mathbf{V}_{se}^{abc} \end{bmatrix} \tag{5.101}$$

where the series transformer may consist of three separate single-phase units or three single-phase units with the secondary sides being delta-connected. For the former, the submatrices are similar to type 1 transformer in Table 5.4 while for the later, the submatrices are similar to type 3 transformer in Table 5.4. $\mathbf{I}_{ij}^{abc}$ and $\mathbf{I}_{se}^{abc}$ are the current vectors of the primary and secondary windings of the series converter transformer, respectively and given by:

$$\mathbf{I}_{ij}^{abc} = [\mathbf{I}_{ij}^a, \mathbf{I}_{ij}^b, \mathbf{I}_{ij}^c]^T \tag{5.102}$$

$$\mathbf{I}_{se}^{abc} = [\mathbf{I}_{se}^a, \mathbf{I}_{se}^b, \mathbf{I}_{se}^c]^T \tag{5.103}$$

The voltage vectors $\mathbf{V}_j^{abc}$ and $\mathbf{V}_{se}^{abc}$ are:

$$\mathbf{V}_j^{abc} = [V_j^a, V_j^b, V_j^c]^T \tag{5.104}$$

$$\mathbf{V}_{se}^{abc} = [V_{se}^a, V_{se}^b, V_{se}^c]^T \tag{5.105}$$

The active and reactive power flows of the primary side of the series converter transformer leaving bus i are:

$$\begin{aligned}
P_{ij}^p &= \text{Re}[\mathbf{V}_i^p (\mathbf{I}_{ij}^p)^*] \\
&= \sum_{m=a,b,c} V_i^p V_i^m (Gse_{pp}^{pm} \cos(\theta_i^p - \theta_i^m) + Bse_{pp}^{pm} \sin(\theta_i^p - \theta_i^m)) \\
&\quad - \sum_{m=a,b,c} V_i^p V_j^m (Gse_{pp}^{pm} \cos(\theta_i^p - \theta_j^m) + Bse_{pp}^{pm} \sin(\theta_i^p - \theta_j^m)) \\
&\quad + \sum_{m=a,b,c} V_i^p V_{se}^m (Gse_{ps}^{pm} \cos(\theta_i^p - \theta_{se}^m) + Bse_{ps}^{pm} \sin(\theta_i^p - \theta_{se}^m))
\end{aligned} \tag{5.106}$$

$$\begin{aligned}
Q_{ij}^p &= \text{Im}[\mathbf{V}_i^p (\mathbf{I}_{ij}^p)^*] \\
&= \sum_{m=a,b,c} V_i^p V_i^m (Gse_{pp}^{pm} \sin(\theta_i^p - \theta_i^m) - Bse_{pp}^{pm} \cos(\theta_i^p - \theta_i^m)) \\
&\quad - \sum_{m=a,b,c} V_i^p V_j^m (Gse_{pp}^{pm} \sin(\theta_i^p - \theta_j^m) - Bse_{pp}^{pm} \cos(\theta_i^p - \theta_j^m)) \\
&\quad + \sum_{m=a,b,c} V_i^p V_{se}^m (Gsh_{ps}^{pm} \sin(\theta_i^p - \theta_{se}^m) - Bsh_{ps}^{pm} \cos(\theta_i^p - \theta_{se}^m))
\end{aligned} \tag{5.107}$$

where $p, m = a, b, c$. $Gse_{pp}^{pm} + jBse_{pp}^{pm} \in \mathbf{Yse}_{pp}^{abc}$, and $Gse_{ps}^{pm} + jBse_{ps}^{pm} \in \mathbf{Yse}_{ps}^{abc}$.

The active and reactive power flows of the primary side of the series converter transformer leaving bus j are:

$$\begin{aligned}
P_{ji}^p &= \text{Re}[\mathbf{V}_j^p (\mathbf{I}_{ji}^p)^*] \\
&= \sum_{m=a,b,c} V_j^p V_j^m (Gse_{pp}^{pm} \cos(\theta_j^p - \theta_j^m) + Bse_{pp}^{pm} \sin(\theta_j^p - \theta_j^m)) \\
&\quad - \sum_{m=a,b,c} V_j^p V_i^m (Gse_{pp}^{pm} \cos(\theta_j^p - \theta_i^m) + Bse_{pp}^{pm} \sin(\theta_j^p - \theta_i^m)) \\
&\quad - \sum_{m=a,b,c} V_j^p V_{se}^m (Gse_{ps}^{pm} \cos(\theta_j^p - \theta_{se}^m) + Bse_{ps}^{pm} \sin(\theta_j^p - \theta_{se}^m))
\end{aligned} \tag{5.108}$$

$$\begin{aligned}
Q_{ji}^p &= \text{Im}[\mathbf{V}_j^p (\mathbf{I}_{ji}^p)^*] \\
&= \sum_{m=a,b,c} V_j^p V_j^m (Gse_{pp}^{pm} \sin(\theta_j^p - \theta_j^m) - Bse_{pp}^{pm} \cos(\theta_j^p - \theta_j^m)) \\
&\quad - \sum_{m=a,b,c} V_j^p V_i^m (Gse_{pp}^{pm} \sin(\theta_j^p - \theta_i^m) - Bse_{pp}^{pm} \cos(\theta_j^p - \theta_i^m)) \\
&\quad - \sum_{m=a,b,c} V_j^p V_{se}^m (Gsh_{ps}^{pm} \sin(\theta_j^p - \theta_{se}^m) - Bsh_{ps}^{pm} \cos(\theta_j^p - \theta_{se}^m))
\end{aligned} \tag{5.109}$$

Similarly, the active and reactive power flows at the secondary side of the series converter transformer are given by:

$$\begin{aligned}
P_{se}^p &= \text{Re}[V_{se}^p (I_{se}^p)^*] \\
&= \sum_{m=a,b,c} V_{se}^p V_{se}^m (G_{ss}^{pm} \cos(\theta_{se}^p - \theta_{se}^m) + B_{ss}^{pm} \sin(\theta_{se}^p - \theta_{se}^m)) \\
&+ \sum_{m=a,b,c} V_{se}^p V_i^m (G_{sp}^{pm} \cos(\theta_{se}^p - \theta_i^m) + B_{sp}^{pm} \sin(\theta_{se}^p - \theta_i^m)) \\
&- \sum_{m=a,b,c} V_{se}^p V_j^m (G_{sp}^{pm} \cos(\theta_{se}^p - \theta_j^m) + B_{sp}^{pm} \sin(\theta_{se}^p - \theta_j^m))
\end{aligned} \tag{5.110}$$

$$\begin{aligned}
Q_{se}^p &= \text{Im}[V_{sh}^p (I_{sh}^p)^*] \\
&= \sum_{m=a,b,c} V_{se}^p V_{se}^m (G_{ss}^{pm} \sin(\theta_{se}^p - \theta_{se}^m) - B_{ss}^{pm} \cos(\theta_{se}^p - \theta_{se}^m)) \\
&+ \sum_{m=a,b,c} V_{se}^p V_i^m (G_{sp}^{pm} \sin(\theta_{se}^p - \theta_i^m) - B_{sp}^{pm} \cos(\theta_{se}^p - \theta_i^m)) \\
&- \sum_{m=a,b,c} V_{se}^p V_j^m (G_{sp}^{pm} \sin(\theta_{se}^p - \theta_j^m) - B_{sp}^{pm} \cos(\theta_{se}^p - \theta_j^m))
\end{aligned} \tag{5.111}$$

5.4.3.3 Active Power Balance of the UPFC

The active power exchange among the converters via the DC link should be balanced at any instant, which is described by:

$$P_{\Sigma} = \sum_{p=a,b,c} P_{sh}^p + \sum_{p=a,b,c} P_{se}^p + P_{loss} = 0 \tag{5.112}$$

where P_{sh}^p and P_{se}^p are defined by (5.99) and (5.110), respectively. P_{loss} represents losses in converter circuits. Each converter losses consist of two terms. The first term is proportional to its ac terminal current squared, and the second term is a constant. The former may be represented by an equivalent resistance, and can be included into its coupling transformer impedance. The latter of all the converters can be combined and represented by P_{loss} .

5.4.4 Symmetrical Components Control Model for Three-Phase UPFC

The symmetrical components control assumes that both the three-phase shunt and series converter injects three-phase balanced voltages. Basically such a control is applicable to a three-phase UPFC with any series and shunt transformer connection types. In principle, the control is identical to that of the positive-sequence control of the UPFC in conventional positive-sequence power flow analysis when the three-phase network and loads are balanced.

5.4.4.1 PQ Flow Control by the Series Converter

The injected three-phase series voltages V_{se}^p ($p=a, b, c$) should be balanced, this means that the three-phase voltages are identical in magnitude while their phase angles have 120° displacement between them. The balanced three-phase voltages may be represented by the constraints as follows:

$$\Delta V_{se}^1_{Re} = \text{Re}(V_{se}^a - V_{se}^b e^{j120^\circ}) = 0 \quad (5.113)$$

$$\Delta V_{se}^1_{Im} = \text{Im}(V_{se}^a - V_{se}^b e^{j120^\circ}) = 0 \quad (5.114)$$

$$\Delta V_{se}^2_{Re} = \text{Re}(V_{se}^a - V_{se}^c e^{j240^\circ}) = 0 \quad (5.115)$$

$$\Delta V_{se}^2_{Im} = \text{Im}(V_{se}^a - V_{se}^c e^{j240^\circ}) = 0 \quad (5.116)$$

For the three-phase UPFC, the series converter can be used to control the total three-phase active and reactive power flows of the transmission line. The control constraints are given by:

$$P_{ji}^\Sigma - P_{spec}^\Sigma_{ji} = 0 \quad (5.117)$$

$$Q_{ji}^\Sigma - Q_{spec}^\Sigma_{ji} = 0 \quad (5.118)$$

where $P_{spec}^\Sigma_{ji}$ and $Q_{spec}^\Sigma_{ji}$ are the specified total three-phase active and reactive power flow control references, respectively. P_{ji}^Σ and Q_{ji}^Σ are the actual total three-phase active and reactive power flows, respectively and given by:

$$P_{ji}^\Sigma = \sum_{p=a,b,c} P_{ji}^p \quad (5.119)$$

$$Q_{ji}^\Sigma = \sum_{p=a,b,c} Q_{ji}^p \quad (5.120)$$

where P_{ji}^p and Q_{ji}^p are defined by (5.108) and (5.109), respectively.

5.4.4.2 Voltage Control by the Shunt Converter

For the symmetrical components control model, it is assumed that the injected three-phase shunt voltages V_{sh}^p ($p=a, b, c$) should be balanced. The balanced three-phase voltages may be represented by the following constraints:

$$\Delta V_{sh}^1_{Re} = \text{Re}(V_{sh}^a - V_{sh}^b e^{j120^\circ}) = 0 \quad (5.121)$$

$$\Delta V_{sh}^1_{Im} = \text{Im}(V_{sh}^a - V_{sh}^b e^{j120^\circ}) = 0 \quad (5.122)$$

$$\Delta V_{sh}^2_{Re} = \text{Re}(V_{sh}^a - V_{sh}^c e^{j240^\circ}) = 0 \quad (5.123)$$

$$\Delta V_{sh}^2 = \text{Im}(\mathbf{V}_{sh}^a - \mathbf{V}_{sh}^c e^{j240^\circ}) = 0 \quad (5.124)$$

For the three-phase UPFC, it may be used to control the positive-sequence voltage at bus i :

$$V_i^1 - V_{spec_i}^1 = 0 \quad (5.125)$$

where V_i^1 is the actual positive-sequence voltage at bus i and can be represented by phase voltages V_i^a , V_i^b and V_i^c while $V_{spec_i}^1$ is the positive-sequence voltage control reference.

5.4.4.3 Transformer Models

For this control model of the UPFC, the shunt converter transformer may be of any of the connection types shown in Table 5.4 while the secondary sides of the three single-phase transformers may be delta-connected or may be separated as shown in Fig. 5.7.

5.4.4.4 Modeling of Three-Phase UPFC in Newton Power Flow

Basically, the three-phase UPFC has twelve operating and control constraints (5.112) – (5.125). In addition, the state variables such as V_{se}^p and V_{sh}^p may be constrained by the converter voltage ratings, and the currents through the converter should be within its current ratings.

For the symmetrical components control model of the UPFC, the Newton equation including six power mismatches at buses i, j and twelve operating and control mismatches may be written as:

$$\mathbf{J}\Delta\mathbf{X} = -\mathbf{F}(\mathbf{X}) \quad (5.126)$$

where

$\Delta\mathbf{X}$ - the incremental vector of state variables, and $\Delta\mathbf{X} = [\Delta\mathbf{X}_{upfc}, \Delta\mathbf{X}_{sys}]^T$

$\Delta\mathbf{X}_{sys} = [\Delta\theta_i^p, \Delta V_i^p, \Delta\theta_j^p, \Delta V_j^p]^T$ - the incremental vector of bus voltage angles and magnitudes.

$\Delta\mathbf{X}_{upfc} = [\Delta\theta_{se}^p, \Delta V_{se}^p, \Delta\theta_{sh}^p, \Delta V_{sh}^p]^T$ - the incremental vector of the UPFC state variables.

$\mathbf{F}(\mathbf{X}) = [\mathbf{F}_{upfc}, \mathbf{F}_{sys}]^T$ - bus power and the UPFC operating and control mismatch vector.

$\mathbf{F}_{sys} = [\Delta P_i^p, \Delta Q_i^p, \Delta P_j^p, \Delta Q_j^p]^T$ - power mismatch vector.

$\mathbf{F}_{upfc} = [P_\Sigma, V_i^1 - V_{spec_i}^1, \Delta V_{sh}^1, \Delta V_{sh}^2, \Delta V_{sh}^2, \Delta V_{sh}^2, P_{ji}^\Sigma - P_{spec_{ji}}^\Sigma, Q_{ji}^\Sigma - Q_{spec_{ji}}^\Sigma, \Delta V_{se}^1, \Delta V_{se}^1, \Delta V_{se}^2, \Delta V_{se}^2]^T$ - the UPFC operating and control mismatches

$$\mathbf{J} = \frac{\partial \mathbf{F}(\mathbf{X})}{\partial \mathbf{X}} \text{ - System Jacobian matrix.}$$

5.4.5 General Three-Phase Control Model for Three-Phase UPFC

For the general control model of the three-phase UPFC, the series converter can be used to control the six independent active and reactive power flows of the transmission line while the shunt converter can be used to control the three-phase voltages at the shunt bus.

5.4.5.1 PQ Flow Control by the Series Converter

The six independent active and reactive power control constraints of the series control of the UPFC are:

$$P_{ji}^p - P_{spec}^p = 0 \quad (p = a, b, c) \quad (5.127)$$

$$Q_{ji}^p - Q_{spec}^p = 0 \quad (p = a, b, c) \quad (5.128)$$

where P_{spec}^p , Q_{spec}^p are the specified active and reactive power flow control references of phase p .

5.4.5.2 Voltage Control by the Shunt Converter

For the general control model of the three-phase UPFC, it may be used to control three-phase voltages at bus i . The control constraints are given by:

$$V_i^p - V_{spec}^p = 0 \quad (p = a, b, c) \quad (5.129)$$

where V_i^p is the actual phase voltage at bus i while V_{spec}^p is the phase voltage control reference.

5.4.5.3 Operating Constraints of the Shunt Transformer

In this control model, it is assumed that the zero-sequence voltage component at the secondary side of the shunt transformer is zero:

$$\operatorname{Re}\left(\sum_{p=a,b,c} V_{sh}^p\right) = \sum_{p=a,b,c} V_{sh}^p \cos \theta_{sh}^p = 0 \quad (5.130)$$

$$\operatorname{Im}\left(\sum_{p=a,b,c} V_{sh}^p\right) = \sum_{p=a,b,c} V_{sh}^p \sin \theta_{sh}^p = 0 \quad (5.131)$$

5.4.5.4 Transformer Models

For this control model of the UPFC, the shunt converter transformer may be of any of the connection types as shown in Table 5.4 while the series converter is connected with the system via three separate single-phase transformers where the secondary sides of the transformers are not connected.

5.4.5.5 Modeling of Three-Phase UPFC in Newton Power Flow

For the general three-phase control model of the UPFC, the Newton equation including six power mismatches at buses i, j and twelve operating and control mismatches (5.112), (5.127)–(5.131) may be written as:

$$\mathbf{J}\Delta\mathbf{X} = -\mathbf{F}(\mathbf{X}) \quad (5.132)$$

where

$\Delta\mathbf{X}$ - the incremental vector of state variables, and $\Delta\mathbf{X} = [\Delta\mathbf{X}_{upfc}, \Delta\mathbf{X}_{sys}]^T$

$\Delta\mathbf{X}_{sys} = [\Delta\theta_i^p, \Delta V_i^p, \Delta\theta_j^p, \Delta V_j^p]^T$ - the incremental vector of bus voltage angles and magnitudes.

$\Delta\mathbf{X}_{upfc} = [\Delta\theta_{se}^p, \Delta V_{se}^p, \Delta\theta_{sh}^p, \Delta V_{sh}^p]^T$ - the incremental vector of the UPFC state variables.

$\mathbf{F}(\mathbf{X}) = [\mathbf{F}_{upfc}, \mathbf{F}_{sys}]^T$ - bus power and the UPFC operating and control mismatch vector.

$\mathbf{F}_{sys} = [\Delta P_i^p, \Delta Q_i^p, \Delta P_j^p, \Delta Q_j^p]^T$ - power mismatch vector.

$\mathbf{F}_{upfc} = [P_\Sigma, V_i^p - V_{spec}^p, P_{ji}^p - P_{spec}^p, Q_{ji}^p - Q_{spec}^p, \text{Re}(\sum_{p=a,b,c} V_{sh}^p), \text{Im}(\sum_{p=a,b,c} V_{sh}^p)]^T$

- the UPFC operating and control mismatches

$\mathbf{J} = \frac{\partial \mathbf{F}(\mathbf{X})}{\partial \mathbf{X}}$ - System Jacobian matrix.

5.4.6 Hybrid Control Model for Three-Phase UPFC

In contrast to the general control model presented in the previous section, the hybrid control model assumes:

- the positive-sequence voltage at bus i and the active and reactive power flows of each phase of the transmission line are controlled;
- the shunt converter injects three-phase balanced voltages only.

5.4.6.1 PQ Flow Control by the Series Converter

For the hybrid control model, the phase series voltages V_{se}^p ($p=a, b, c$) is injected to control the active and reactive power flows of that phase. The control constraints are given by:

$$P_{ji}^p - P_{spec}^p = 0 \quad (5.133)$$

$$Q_{ji}^p - Q_{spec}^p = 0 \quad (5.134)$$

where P_{spec}^p and Q_{spec}^p ($p=a, b, c$) are the specified phase active and reactive power flow control references, respectively. P_{ji}^p and Q_{ji}^p ($p=a, b, c$) are the actual phase active and reactive power flows, respectively.

5.4.6.2 Voltage Control by the Shunt Converter

For the hybrid control model, the injected three-phase shunt voltages V_{sh}^p ($p=a, b, c$) should be balanced. We have:

$$\Delta V_{sh}^1_{Re} = \text{Re}(V_{sh}^a - V_{sh}^b e^{j120^\circ}) = 0 \quad (5.135)$$

$$\Delta V_{sh}^1_{Im} = \text{Im}(V_{sh}^a - V_{sh}^b e^{j120^\circ}) = 0 \quad (5.136)$$

$$\Delta V_{sh}^2_{Re} = \text{Re}(V_{sh}^a - V_{sh}^c e^{j240^\circ}) = 0 \quad (5.137)$$

$$\Delta V_{sh}^2_{Im} = \text{Im}(V_{sh}^a - V_{sh}^c e^{j240^\circ}) = 0 \quad (5.138)$$

Assuming that the shunt converter is used to control the positive-sequence voltage at bus i , the control constraint is given by:

$$V_i^1 - V_{spec}^1 = 0 \quad (5.139)$$

where V_i^1 is the actual positive-sequence voltage at bus i while V_{spec}^1 is the positive-sequence voltage control reference.

5.4.6.3 Transformer Models

For this control model of the UPFC, the shunt converter transformer may be of any of the connection types as shown in Table 5.4 while the series converter is connected with the system via three separate single-phase transformers where the secondary sides of the transformers are not connected.

5.4.6.4 Modeling of Three-Phase UPFC in the Newton Power Flow

Basically, the hybrid UPFC control model has eleven control constraints given by (5.133)-(5.139), and the power balance constraint given by (5.112).

For the hybrid UPFC control model, the Newton equation including six power mismatches at buses i, j and twelve operating and control mismatches may be written as:

$$\mathbf{J}\Delta\mathbf{X} = -\mathbf{F}(\mathbf{X}) \quad (5.140)$$

where

$\Delta\mathbf{X}$ - the incremental vector of state variables, and $\Delta\mathbf{X} = [\Delta\mathbf{X}_{upfc}, \Delta\mathbf{X}_{sys}]^T$

$\Delta\mathbf{X}_{sys} = [\Delta\theta_i^p, \Delta V_i^p, \Delta\theta_j^p, \Delta V_j^p]^T$ - the incremental vector of bus voltage angles and magnitudes.

$\Delta\mathbf{X}_{upfc} = [\Delta\theta_{se}^p, \Delta V_{se}^p, \Delta\theta_{sh}^p, \Delta V_{sh}^p]^T$ - the incremental vector of the UPFC state variables.

$\mathbf{F}(\mathbf{X}) = [\mathbf{F}_{upfc}, \mathbf{F}_{sys}]^T$ - bus power and the UPFC operating and control mismatch vector.

$\mathbf{F}_{sys} = [\Delta P_i^p, \Delta Q_i^p, \Delta P_j^p, \Delta Q_j^p]^T$ - power mismatch vector.

$\mathbf{F}_{upfc} = [P_\Sigma, V_i^1 - Vspec_i^1, \Delta Vsh_{Re}^1, \Delta Vsh_{Im}^1, \Delta Vsh_{Re}^2, \Delta Vsh_{Im}^2, P_{ji}^p - Pspec_{ji}^p, Q_{ji}^p - Qspec_{ji}^p]^T$ - the UPFC operating and control mismatches

$\mathbf{J} = \frac{\partial \mathbf{F}(\mathbf{X})}{\partial \mathbf{X}}$ - System Jacobian matrix

5.4.7 Numerical Examples

In this section, numerical results are presented for a 5-bus system and the IEEE 118-bus system. The 5 bus three-phase system is shown in Fig. 5.8 in the Appendix of this chapter, while the system parameters are listed in Table 5.11 - Table 5.14. In order to make simulations on the IEEE 118-bus system realistic, a Delta/Wye-G transformer is inserted between each generator and its terminal bus.

In the following tests, a convergence tolerance of $1.0\text{e-}12$ p.u. (or $1.0\text{e-}10$ MW/MVAr) for maximal absolute bus power mismatches and power flow control mismatches is used. In order to simplify the following presentation, the Symmetrical Components Control Model proposed in section 5.4.4 is referred to Model I, the General Control Model in section 5.4.5 is referred to Model II while the Hybrid Control Model proposed in section 5.4.6 is referred to Model III.

5.4.7.1 Results for the 5-Bus System

In order to validate the three-phase control models of the UPFC, two cases are carried out under the balanced network and load condition:

Case I: Well transposed transmission lines and the whole system with balanced load. A UPFC is inserted between the receiving end of line 1-3 and bus 3. Suppose the receiving end bus of line 1-3 is now referred to bus 3'. The

whole system is represented only by the positive-sequence network and load. The power flow is solved by the single-phase positive-sequence power flow.

Case 2: Well transposed transmission lines and the whole system with balanced load. A UPFC is inserted between the receiving end of line 1-3 and bus 3. The power flow is solved by the three-phase power flow.

The single-phase power flow control reference of the UPFC is $7.0+j1.6$ p.u. while the total three-phase power flow control reference is $21.0+j4.8$ p.u. The voltage control reference is 1.0 p.u. The power flow solutions of case 1 and case 2 are shown in Table 5.5 and Table 5.6

Table 5.5. Power flow solutions for the balanced 5 bus system by single-phase and three-phase power flow algorithms

Case 1			Case 2		
Bus No.	V_i (p.u.)	θ_i (deg)	Bus No.	V_i^a (p.u.)	θ_i^a (deg)
1	1.0107	-3.02	1	1.0107	26.98
2	1.0196	-1.43	2	1.0196	28.57
3	1.0000	0.56	3	1.0000	30.56
4	1.0450	0.00	4	1.0450	0.00
5	1.0610	2.33	5	1.0610	2.33

Table 5.6. UPFC solutions on the 5 bus system by single-phase and three-phase power flow algorithms

Case 1		Case 2			
Shunt converter	Series converter	Control models	Shunt converter transformer types	Shunt converter	Series converter
$\theta_{sh} = 0.66^\circ$ $V_{sh} = 0.9886$ p.u.	$\theta_{se} = 67.78^\circ$ $V_{se} = 0.2982$ p.u.	I, II, III	1, 2, 4, 6, 7	$\theta_{sh}^a = 30.66^\circ$ $V_{sh}^a = 0.9886$ p.u.	$\theta_{se}^a = 97.78^\circ$ $V_{se}^a = 0.2982$ p.u.
		I, II, III	3, 5	$\theta_{sh}^a = 0.66^\circ$ $V_{sh}^a = 0.9886$ p.u.	$\theta_{se}^a = 97.78^\circ$ $V_{se}^a = 0.2982$ p.u.
		I, II, III	8, 9	$\theta_{sh}^a = 60.66^\circ$ $V_{sh}^a = 0.9886$ p.u.	$\theta_{se}^a = 97.78^\circ$ $V_{se}^a = 0.2982$ p.u.

From Table 5.5, it can be found that the bus voltages of the two cases are identical except the 30 degree angle shifting of the voltage angles from the three-phase power flow solution caused by the Wye-G/Delta transformers. For case 2, with the different UPFC models and the different UPFC shunt transformer types, the power flow solutions shown in Table 5.5 are the same except that some of the UPFC injected voltages in Table 5.6 have 30 or 60 degree shifting caused by the Wye/Delta and Delta/Wye transformers. The computation results indicate the validity of the UPFC models proposed. The test results shown in Table 5.5 and Table 5.6 imply that positive-sequence representation of a power system is normally sufficient when the system is balanced.

In order to investigate the behavior and control performance of the three UPFC control models proposed, case studies are carried out for the 5-bus system when the network is unbalanced and there is unbalanced load at bus 3. The power flow and voltage control references of the UPFC are the same to those of the balanced case. The system data are given by Appendix while the test results are given by Table 5.7 to Table 5.9. From these tables it can be found:

1. The power flow solutions with the different UPFC control models are not the same when the system is unbalanced. This implies that under unbalanced conditions, three-phase modeling of the system is needed and proper modeling of three-phase UPFC and its controls should be considered.
2. The power flow solutions with the same UPFC control model and the different shunt converter transformer connection types are not the same when the system is unbalanced. This indicates that appropriate modeling of UPFC transformers is needed when the system is unbalanced.

Table 5.7. Power flow solutions for the unbalanced 5 bus system with UPFC Model I

Case No.	3	4
Shunt converter transformer type	3	8
Shunt bus	$V_3^a = 0.9961$ $\theta_3^a = 30.35^\circ$ $V_3^b = 1.0106$ $\theta_3^b = -89.82^\circ$ $V_3^c = 0.9933$ $\theta_3^c = 150.17^\circ$	$V_3^a = 0.9994$ $\theta_3^a = 29.59^\circ$ $V_3^b = 1.0253$ $\theta_3^b = -89.71^\circ$ $V_3^c = 0.9854$ $\theta_3^c = 150.78^\circ$
Shunt converter	$V_{sh}^a = 0.9914$ $\theta_{sh}^a = 0.32^\circ$ $V_{sh}^b = 0.9914$ $\theta_{sh}^b = -119.68^\circ$ $V_{sh}^c = 0.9914$ $\theta_{sh}^c = 120.32^\circ$	$V_{sh}^a = 0.9915$ $\theta_{sh}^a = 60.30^\circ$ $V_{sh}^b = 0.9915$ $\theta_{sh}^b = -59.70^\circ$ $V_{sh}^c = 0.9915$ $\theta_{sh}^c = 180.30^\circ$
Series converter	$V_{se}^a = 0.2667$ $\theta_{se}^a = 99.45^\circ$ $V_{se}^b = 0.2667$ $\theta_{se}^b = -20.55^\circ$ $V_{se}^c = 0.2667$ $\theta_{se}^c = -140.55^\circ$	$V_{se}^a = 0.2667$ $\theta_{se}^a = 99.37^\circ$ $V_{se}^b = 0.2667$ $\theta_{se}^b = -20.63^\circ$ $V_{se}^c = 0.2667$ $\theta_{se}^c = -140.63^\circ$
Number of iterations	6	6

Table 5.8. Power flow solutions for the unbalanced 5 bus system with UPFC Model II

Case No.	5	6
Shunt converter transformer type	3	8
Shunt bus	$V_3^a = 1.0000 \quad \theta_3^a = 30.54^\circ$ $V_3^b = 1.0000 \quad \theta_3^b = -90.03^\circ$ $V_3^c = 1.0000 \quad \theta_3^c = 150.48^\circ$	$V_3^a = 1.0000 \quad \theta_3^a = 32.21^\circ$ $V_3^b = 1.0000 \quad \theta_3^b = -91.56^\circ$ $V_3^c = 1.0000 \quad \theta_3^c = 150.54^\circ$
Shunt converter	$V_{sh}^a = 0.9871 \quad \theta_{sh}^a = 0.19^\circ$ $V_{sh}^b = 0.9873 \quad \theta_{sh}^b = -119.32^\circ$ $V_{sh}^c = 0.9945 \quad \theta_{sh}^c = 120.41^\circ$	$V_{sh}^a = 1.0099 \quad \theta_{sh}^a = 60.82^\circ$ $V_{sh}^b = 0.9850 \quad \theta_{sh}^b = -60.68^\circ$ $V_{sh}^c = 0.9750 \quad \theta_{sh}^c = 181.34^\circ$
Series converter	$V_{se}^a = 0.3029 \quad \theta_{se}^a = 93.93^\circ$ $V_{se}^b = 0.3029 \quad \theta_{se}^b = -8.23^\circ$ $V_{se}^c = 0.3029 \quad \theta_{se}^c = -144.83^\circ$	$V_{se}^a = 0.3261 \quad \theta_{se}^a = 96.57^\circ$ $V_{se}^b = 0.1885 \quad \theta_{se}^b = -8.99^\circ$ $V_{se}^c = 0.3193 \quad \theta_{se}^c = -145.09^\circ$
Number of iterations	6	7

Table 5.9. Power flow solutions for the unbalanced 5 bus system with UPFC Model III

Case No.	7	8
Shunt converter transformer type	3	8
Shunt bus	$V_3^a = 1.0038 \quad \theta_3^a = 30.58^\circ$ $V_3^b = 0.9978 \quad \theta_3^b = -90.18^\circ$ $V_3^c = 0.9984 \quad \theta_3^c = 150.69^\circ$	$V_3^a = 0.9848 \quad \theta_3^a = 31.47^\circ$ $V_3^b = 0.9921 \quad \theta_3^b = -91.53^\circ$ $V_3^c = 1.0240 \quad \theta_3^c = 151.26^\circ$
Shunt converter	$V_{sh}^a = 0.9896 \quad \theta_{sh}^a = 0.46^\circ$ $V_{sh}^b = 0.9896 \quad \theta_{sh}^b = -119.54^\circ$ $V_{sh}^c = 0.9896 \quad \theta_{sh}^c = 120.46^\circ$	$V_{sh}^a = 0.9894 \quad \theta_{sh}^a = 60.52^\circ$ $V_{sh}^b = 0.9894 \quad \theta_{sh}^b = -59.48^\circ$ $V_{sh}^c = 0.9894 \quad \theta_{sh}^c = 180.52^\circ$
Series converter	$V_{se}^a = 0.3048 \quad \theta_{se}^a = 93.44^\circ$ $V_{se}^b = 0.2117 \quad \theta_{se}^b = -7.87^\circ$ $V_{se}^c = 0.3188 \quad \theta_{se}^c = -144.33^\circ$	$V_{se}^a = 0.3108 \quad \theta_{se}^a = 97.83^\circ$ $V_{se}^b = 0.1885 \quad \theta_{se}^b = -7.21^\circ$ $V_{se}^c = 0.3388 \quad \theta_{se}^c = -147.39^\circ$
Number of iterations	6	7

5.4.7.2 Results for the Modified IEEE 118-Bus System

Further tests are carried out on the modified IEEE 118-bus system, which are as follows:

- Case 9:* Well-transposed transmission lines and the system with unbalanced load at bus 45 with $0.73+j0.22$ p.u., $0.53+j0.22$ p.u., $0.23+j0.22$ p.u. for phase a , b , c loading, respectively, and unbalanced load at bus 78 with $0.51+j0.26$ p.u., $0.71+j0.26$ p.u., $0.91+j0.26$ p.u. for phase a , b , c loading, respectively.
- Case 10:* As for case 9, there are two UPFCs installed on the transmission lines 30-38 and 68-81. The control model I is used for the two UPFC.
- Case 11:* Similar to case 10, but the control model II is used for the two UPFC.
- Case 12:* Similar to case 10, but the control model III is used for the two UPFC.
- Case 13:* Similar to case 10, but the control model I is used for the UPFC on line 30-38 and the control model II is used for the UPFC on line 68-81.
- Case 14:* Similar to case 10, but the control model I is used for the UPFC on line 30-38 and the control model III is used for the UPFC on line 68-81.
- Case 15:* Similar to case 10, but the control model II is used for the UPFC on line 30-38 and the control model I is used for the UPFC on line 68-81.
- Case 16:* Similar to case 10, but the control model II is used for the UPFC on line 30-38 and the control model III is used for the UPFC on line 68-81.
- Case 17:* Similar to case 10, but the control model III is used for the UPFC on line 30-38 and the control model I is used for the UPFC on line 68-81.
- Case 18:* Similar to case 10, but the control model III is used for the UPFC on line 30-38 and the control model II is used for the UPFC on line 68-81.

In cases 9 - 18, the active power control references of the UPFC are 140% of the base case power flows, respectively. It is assumed that (a) three separate series transformer units are used for each UPFC; (b) the shunt transformer of the UPFC on line 30-38 is a Wye-G/Delta three-phase transformer while the shunt transformer of the UPFC on line 68-81 is a Delta/Delta three-phase transformer. The test results are shown in Table 5.10. For all the cases above for the modified IEEE 118-bus system, the power flow algorithm can converge within 8 iterations.

Table 5.10. Results for the modified IEEE 118-bus system

Case No.	Number of iterations
9	6
10-18	8

5.5 Three-Phase Newton OPF in Polar Coordinates

Mathematically, as an example the objective function of a three-phase OPF may be minimizing the total operating cost as follows:

$$\text{Minimize } f(x) = \sum_i^{Ng} (\alpha_i * Pg_i^2 + \beta_i * Pg_i + \gamma_i) \quad (5.141)$$

while subject to the following constraints:

Nonlinear equality constraints:

$$\begin{aligned} \Delta P_i^p &= -Pd_i^p - V_i^p \sum_{j \in i} \sum_{m=a,b,c} V_j^m (G_{ij}^{pm} \cos \theta_{ij}^{pm} + B_{ij}^{pm} \sin \theta_{ij}^{pm}) \\ (p &= a, b, c, \text{ and } i=1, 2, \dots, N) \end{aligned} \quad (5.142)$$

$$\begin{aligned} \Delta Q_i^p &= -Qd_i^p - V_i^p \sum_{j \in i} \sum_{m=a,b,c} V_j^m (G_{ij}^{pm} \sin \theta_{ij}^{pm} - B_{ij}^{pm} \cos \theta_{ij}^{pm}) \\ (p &= a, b, c, \text{ and } i=1, 2, \dots, N) \end{aligned} \quad (5.143)$$

$$\begin{aligned} \Delta P g_i &= -P g_i \\ &- \sum_{p=a,b,c} \sum_{m=a,b,c} [V_i^p V_i^m (G g_i^{pm} \cos \theta_i^{pm} + B g_i^{pm} \sin \theta_i^{pm}) \\ &+ \sum_{p=a,b,c} \sum_{m=a,b,c} [V_i^p E_i^p (G g_i^{pm} \cos(\theta_i^p - \delta_i^m) + B g_i^{pm} \sin(\theta_i^p - \delta_i^m)) \\ (i &= 1, 2, \dots, Ng) \end{aligned} \quad (5.144)$$

$$\begin{aligned} \Delta Q g_i &= -Q g_i \\ &- \sum_{p=a,b,c} \sum_{m=a,b,c} [V_i^p V_i^m (G g_i^{pm} \sin \theta_i^{pm} - B g_i^{pm} \cos \theta_i^{pm}) \\ &+ \sum_{p=a,b,c} \sum_{m=a,b,c} [V_i^p E_i^p (G g_i^{pm} \sin(\theta_i^p - \delta_i^m) - B g_i^{pm} \cos(\theta_i^p - \delta_i^m)) \\ (i &= 1, 2, \dots, Ng) \end{aligned} \quad (5.145)$$

Inequality constraints:

$$(P_{ij}^p)^2 + (Q_{ij}^p)^2 \leq (S_{ij}^{\max})^2 \quad (5.146)$$

$$P_i^{\min} \leq P g_i \leq P_i^{\max} \quad (i = 1, 2, Ng) \quad (5.147)$$

$$Q_i^{\min} \leq Q g_i \leq Q_i^{\max} \quad (i = 1, 2, Ng) \quad (5.148)$$

$$t_i^{\min} \leq t_i \leq t_i^{\max} \quad (i = 1, 2, Nt) \quad (5.149)$$

$$V_i^{\min} \leq V_i \leq V_i^{\max} \quad (i = 1, 2, N) \quad (5.150)$$

where

$\alpha_i, \beta_i, \gamma_i$ coefficients of production cost functions of generator

ΔP_i^P	bus active power mismatch equations
ΔQ_i^P	bus reactive power mismatch equations
P_{ij}^P	active line power flow
Q_{ij}^P	Reactive line power flow
P_g	the vector of active power generation
Q_g	the vector of reactive power generation
θ_g	the vector of generator internal bus voltage angle
V_g	the vector of generator internal bus voltage magnitude
θ	the vector of bus voltage angle
V	the vector of bus voltage magnitude
t	the vector of transformer tap ratios
$x = [P_g, Q_g, \theta_g, V_g, t, \theta, V]^T$ is the vector of variables	
N	the number of system buses excluding the generator internal buses
N_g	the number of generators
N_t	the number of transformers

The power flows P_{ij}^P and Q_{ij}^P are given by:

$$P_{ij}^P = V_i^P \sum_{m=a,b,c} V_j^m (G_{ij}^{pm} \cos \theta_{ij}^{pm} + B_{ij}^{pm} \sin \theta_{ij}^{pm}) \quad (p = a, b, c) \quad (5.151)$$

$$Q_{ij}^P = V_i^P \sum_{m=a,b,c} V_j^m (G_{ij}^{pm} \sin \theta_{ij}^{pm} - B_{ij}^{pm} \cos \theta_{ij}^{pm}) \quad (p = a, b, c) \quad (5.152)$$

In the three-phase OPF problem of (5.141)-(5.150), the SSSC and UPFC models with the extra equalities and inequalities, which have been presented in previous sections, can be included. The three-phase OPF problem may be solved by the nonlinear interior point methods that have been applied to the conventional OPF problems. With the integration of distributed generation into power networks, a three-phase OPF tool will be required in the operation, control and planning of power networks to ensure the security and reliability.

5.6 Appendix A - Definition of Yg_i

Zg_i is the impedance matrix of a synchronous machine, which is given by:

$$\begin{aligned}
 Zg_i &= T_{120}^{abc} \begin{bmatrix} z_1 & 0 & 0 \\ 0 & z_2 & 0 \\ 0 & 0 & z_0 \end{bmatrix} T_{abc}^{120} \\
 &= \frac{1}{3} \begin{bmatrix} z_0 + z_1 + z_2 & z_0 + az_1 + a^2z_2 & z_0 + a^2z_1 + az_2 \\ z_0 + a^2z_1 + az_2 & z_0 + z_1 + z_2 & z_0 + az_1 + a^2z_2 \\ z_0 + az_1 + a^2z_2 & z_0 + a^2z_1 + az_2 & z_0 + z_1 + z_2 \end{bmatrix}
 \end{aligned} \quad (5.153)$$

where T_{abc}^{120} and T_{120}^{abc} are the transformation matrix of symmetrical components and its inverse matrix, respectively. z_1 , z_2 and z_0 are the positive-, negative-, and zero-sequence impedances of a synchronous machine. $a = e^{j2\pi/3}$.

Yg_i is the admittance matrix of a synchronous machine, which is given by:

$$Yg_i = (Zg_i)^{-1} \quad (5.154)$$

5.7 Appendix B - 5-Bus Test System

The 5 bus three-phase system is shown in Fig. 5.8. The system parameters are listed in Table 5.11 to Table 5.14.

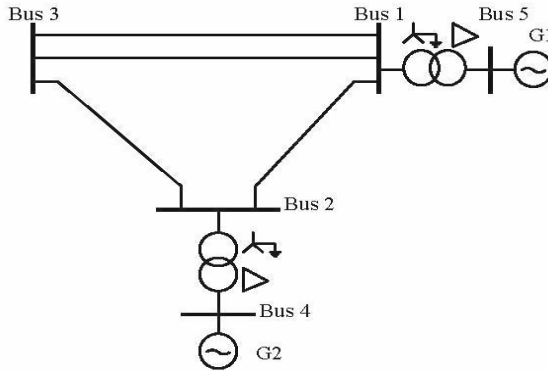


Fig. 5.8. 5-bus test system

Table 5.11. Generator data in p.u.

Generator name	Bus No.	Sequence reactance			Power P	Voltage V
		X_0	X_1	X_2		
G1	5	0.02	0.20	0.04	21.0	1.061
G2	4	0.02	0.20	0.04	slack	1.045

Table 5.12. Transformer data in p.u.

Transformer	T1 & T2
Connection	Wye-G/Delta
Leakage impedance	0.0016+j0.015
Primary tap	1.0
Secondary tap	1.0

Table 5.13. Unbalanced line data for line 1-2, line 1-3 and line 2-3

Series impedance matrix (p.u.)		
Phase a	Phase b	Phase c
0.0066 + j 0.0560	0.0017 + j 0.0270	0.0012 + j 0.0210
	0.0045 + j 0.0470	0.0014 + j 0.0220
		0.0062 + j 0.0610
Shunt admittance matrix (p.u.)		
Phase a	Phase b	Phase c
j 0.150	- j 0.030	- j 0.010
	j 0.250	- j 0.020
		j 0.125

Table 5.14. Load data of the 5-bus system

	Phase a	Phase b	Phase c
Bus 1	0.6 + j 0.3	0.6 + j 0.3	0.6 + j 0.3
Bus 2	2.0 + j 0.8	2.0 + j 0.8	2.0 + j 0.8
Bus 3	6.0 + j 3.0	6.3 + j 2.7	5.7 + j 3.3

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